Detecting Reliable Software Using SPRT: An Order Statistics Approach

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Abstract

To assess the software reliability by statistical means yields efficient results. In this paper, for an effective monitoring of failure process we have opted Sequential Probability Ratio Test (SPRT) over the time between every r[th] failure (r is a natural number >=2) instead of inter-failure times. This paper projects a controlling framework based on order statistics of the cumulative quantity between observations of time domain failure data using mean value function of Logarithmic Poisson Execution Time Model (LPETM). The two unknown parameters can be estimated using the Maximum Likelihood Estimation (MLE).

Keywords

SPRT, Software Quality, Time Domain Data, Order Statistics, LPETM, MLE

I. Introduction

SPRTs are widely used for statistical quality control in manufacturing processes. If r is a natural number (<n), the summations, $\sum_{i=1}^{r} X_i$, $\sum_{i=r+1}^{2r} X_i$, $\sum_{i=2r+1}^{3r} X_i$ etc represent the lapse of time consecutively between every r[th] failure [10]. If (X_1, X_2, …, X_r); (X_{r+1}, X_{r+2}, …, X_{2r}); (X_{2r+1}, X_{2r+2}, …, X_{3r}); etc are regarded as independent samples of size r each then $Y_1=X_1$, $Y_2=$ $Y_2=\sum_{i=1}^{2} X_i$, $Y_3=$ $Y_3=\sum_{i=1}^{3} X_i$, …, $Y_r=$ $Y_r=\sum_{i=1}^{r} X_i$ becomes an ordered sample of size-r representing the time to first failure, time to second failure,..., time to rth failure respectively. Y_r is the highest order statistics in an ordered sample $Y_1<Y_2<...<Y_r$. We know that $[F(x)]^r$ is the cumulative distribution function of r[th] order statistic in a sample of size r for the model F(x). In this paper we make use of ordered samples of size r as time to every rth failure in assessing the reliability of software product.

II. Review of Wald's Sequential Test

An SPRT for homogeneous Poisson processes is described below. Let {N(t),t ≥0} be a homogeneous Poisson process with rate 'λ'. In our case, N(t)=number of failures up to time ' t' and ' λ' is the failure rate (failures per unit time). To avoid the risk of getting wrong answers with statistical tests, assume two small numbers 'α' and 'β', where 'α' is the probability of falsely rejecting the system. That is rejecting the system even if $\lambda \leq \lambda_0$. β is the probability of falsely accepting the system .That is accepting the system even if $\lambda \geq \lambda_1$. With specified choices of λ_0 and λ_1 such that $0 <\lambda_0 <\lambda_1$, the probability of finding N(t) failures in the time span (0,t) with λ_1,λ_0 as the failure rates are respectively given by

$$P_1 = \frac{e^{-\lambda_1 t}\left[\lambda_1 t\right]^{N(t)}}{N(t)!} \qquad (1)$$

$$P_0 = \frac{e^{-\lambda_0 t}\left[\lambda_0 t\right]^{N(t)}}{N(t)!} \qquad (2)$$

The ratio $\frac{P_1}{P_0}$ at any time 't' is considered as a measure of deciding the truth towards λ_0 or λ_1, given a sequence of time instants say $t_1 < t_2 < t_3 < .... < t_K$ and the corresponding realizations. $N(t_1), N(t_2), ...N(t_K)$ of N(t). Simplification of $\frac{P_1}{P_0}$ gives

$$\frac{P_1}{P_0} = \exp(\lambda_0 - \lambda_1)t + \left(\frac{\lambda_1}{\lambda_0}\right)^{N(t)}$$

The decision rule of SPRT is to decide in favor of λ_1, in favor of λ_0 or to continue by observing the number of failures at a later time than 't' according as $\frac{P_1}{P_0}$ is greater than or equal to a constant say A, less than or equal to a constant say B or in between the constants A and B. That is, we decide the given software product as unreliable, reliable or continue the test process with one more observation in failure data, according as

$$\frac{P_1}{P_0} \geq A \qquad (3)$$

$$\frac{P_1}{P_0} \leq B \qquad (4)$$

$$B < \frac{P_1}{P_0} < A \qquad (5)$$

The approximate values of the constants A and B are taken as

$$A \cong \frac{1-\beta}{\alpha}, \quad B \cong \frac{\beta}{1-\alpha}$$

Where 'α' and 'β' are the risk probabilities. If N(t) falls for the first time above the line reject the system as unreliable.

$$N_U(t) = a.t + b_2 \qquad (6)$$

If N(t) falls for the first time below the line accept the system to be reliable.

$$N_L(t) = a.t - b_1 \qquad (7)$$

To continue the test with one more observation on (t, N(t)) as the random graph of [t, N(t)] is between the two linear boundaries given by equations (2.6) and (2.7) where

$$a = \frac{\lambda_1 - \lambda_0}{\log\left(\frac{\lambda_1}{\lambda_0}\right)} \qquad (8)$$

$$b_1 = \frac{\log\left[\frac{1-\alpha}{\beta}\right]}{\log\left(\frac{\lambda_1}{\lambda_0}\right)} \quad (9)$$

$$b_2 = \frac{\log\left[\frac{1-\beta}{\alpha}\right]}{\log\left(\frac{\lambda_1}{\lambda_0}\right)} \quad (10)$$

The parameters α, β, α_0, and α_1 can be chosen in several ways. One way suggested by Stieber (1997) is

$$\lambda_0 = \frac{\lambda.\log(q)}{q-1},\ \lambda_1 = q\frac{\lambda.\log(q)}{q-1}$$

$$where\ q = \frac{\lambda_1}{\lambda_0}$$

If λ_0 and λ_1 are chosen in this way, the slope of N_U (t) and N_L (t) equals λ.

III . Sequential Test for LPETM

We know that for a poisson process the expected value of N(t) = λt called the average number of failures experienced in time 't' which is the mean value function of the Poisson process. Consider a Poisson process with a general function m(t) as its mean value function the probability equation of a such a process is

$$P[N(t)=Y] = \frac{[m(t)]^y}{y!}.e^{-m(t)}, y = 0,1,2,----$$

Depending on the forms of m(t) we get various Poisson processes called NHPP for our model the mean value function is m(t)=a.log(1+bt)

We may write

$$P_1 = \frac{e^{-m_1(t)}.[m_1(t)]^{N(t)}}{N(t)!}$$

$$P_0 = \frac{e^{-m_0(t)}.[m_0(t)]^{N(t)}}{N(t)!}$$

where $m_1(t), m_0(t)$ are values of the mean value function at specified sets of its parameters indicating reliable/unreliable software. The mean value function m(t) of LPETM consists of a pair of parameters a, b with 'a' as a multiplier. Also a, b are positive. Let P_0, P_1 be values of the NHPP at two specifications of b say $b_0, b_1 (b_0 < b_1)$ respectively. So that, for our models m(t) at b_1 is greater than that of at b_0 i.e., $m_0(t)<m_1(t)$.Then the SPRT procedure is as follows:

Accept the system to be reliable $\frac{P_1}{P_0} \le B$

i.e., $\frac{e^{-m_1(t)}.[m_1(t)]^{N(t)}}{e^{-m_0(t)}.[m_0(t)]^{N(t)}} \le B$

i.e.,

$$\text{i.e., } N(t) \le \frac{\log\left(\frac{\beta}{1-\alpha}\right) + m_1(t) - m_0(t)}{\log m_1(t) - \log m_0(t)} \quad (11)$$

Decide the system to be unreliable and reject if $\frac{P_1}{P_0} \ge A$

i.e.,

$$\text{i.e., } N(t) \ge \frac{\log\left(\frac{1-\beta}{\alpha}\right) + m_1(t) - m_0(t)}{\log m_1(t) - \log m_0(t)} \quad (12)$$

Continue the test procedure as long as

$$\frac{\log\left(\frac{\beta}{1-\alpha}\right)+m_1(t)-m_0(t)}{\log m_1(t)-\log m_0(t)} < N(t) < \frac{\log\left(\frac{1-\beta}{\alpha}\right)+m_1(t)-m_0(t)}{\log m_1(t)-\log m_0(t)} \quad (13)$$

Substituting the appropriate expressions of the mean value function m(t)=a.log(1+bt) of LPETM we get the decision rules as follows:

Acceptance region:

$$N(t) \le \frac{\log\left(\frac{\beta}{1-\alpha}\right) + a.\log\left(\frac{1+b_1t}{1+b_0t}\right)}{\log\left(\frac{\log(1+b_1t)}{\log(1+b_0t)}\right)} \quad (14)$$

Rejection region:

$$N(t) \ge \frac{\log\left(\frac{1-\beta}{\alpha}\right) + a.\log\left(\frac{1+b_1t}{1+b_0t}\right)}{\log\left(\frac{\log(1+b_1t)}{\log(1+b_0t)}\right)} \quad (15)$$

Continuation region:

$$\frac{\log\left(\frac{\beta}{1-\alpha}\right) + a.\log\left(\frac{1+b_1t}{1+b_0t}\right)}{\log\left(\frac{\log(1+b_1t)}{\log(1+b_0t)}\right)} < N(t) < \frac{\log\left(\frac{1-\beta}{\alpha}\right) + a.\log\left(\frac{1+b_1t}{1+b_0t}\right)}{\log\left(\frac{\log(1+b_1t)}{\log(1+b_0t)}\right)} \quad (16)$$

It may be significant to note that in the above model the decision rules are exclusively based on the strength of the sequential procedure (α,β) and the values of the mean value functions namely, $m_0(t), m_1(t)$. If the mean value function is linear in 't' passing through origin, that is, m(t) = λt the decision rules become decision lines as described by Stieber (1997). In that sense equations (11), (12), (13) can be regarded as generalizations to the decision procedure of Stieber (1997).The applications of these results are shown in Section V.

IV. Parameter Estimation

In this paper MLE technique is used to estimate the unknown model parameters that maximize the probability of the failure data sets.

The Log likelihood function for the time domain data is,

$$\text{LLF} = \sum_{i=1}^{n} \log[\lambda(s_i)^r] - m(s_n)^r \quad (17)$$

Where λ(t) = $\lambda(t) = \frac{\partial}{\partial t} m(t)$
Since m(t)= a .log(1+bt) we get,
LLF = Log L

$$= \sum_{i=1}^{n} \log\left[\frac{ab}{1+bs_i}\right]^r - a.\log(1+bs_n)^r \quad (18)$$

The unknown parameters a & b of the given LPETM model can be determined using order statistics approach as follows:
To get m(t) value for r^{th} Order Statistics, consider m(t) to the power 'r'
$m(t) = a.\log(1+bt)^r$

The partial derivative of equn (17) w.r.t 'a' and equate it to zero we get,

$$\sum_{i=1}^{n} \frac{\frac{\partial}{\partial a}\left[\frac{ab}{1+bs_i}\right]^r}{\left[\frac{ab}{1+bs_i}\right]^r} - \frac{\partial}{\partial a}(a.\log(1+bs_n))^r = 0$$

$$a^r = \frac{n}{\log(1+bs_n)^r} \quad (19)$$

By Differentiating Eqn.(18) w.r.t 'b' and equate it to zero, we get,

$$g(b) = \sum_{i=1}^{n} \frac{1}{b} - \frac{r*s_n}{\log(1+bs_n)^r(1+bs_n)} - \frac{s_i}{1+bs_i} \quad (20)$$

Differentiating g(b) in Eqn.(20) we get ,

$$g'(b) = \frac{s_n^2 * r}{(1+bs_n)^2 \log(1+bs_n)^r} + \frac{s_n^2 * r^2}{[(1+bs_n)\log(1+bs_n)^r]^2} + \frac{s_i^2}{(1+bs_i)^2} - \frac{1}{b^2} \quad (21)$$

Iterative Newton-Raphson method is used to Solve the equations (19),(20),(21) in order to get the approximated values of a & b for the given failure data sets.

V. Data Analysis Using SPRT

In this section we evaluate the decision rules based on the given mean value function for two different data sets. Based on the estimated value of the parameter 'b', we have chosen the specifications of b_0, b_1 that are to be equidistant such that $b_0 < b < b_1$. The choices are given in the following table.

Table 1: Specifications of b_0, b_1 for Data Set I

Order	Estimate of a	Estimate of b	b_0	b_1
4	1.836407	0.000023	0.000011	0.000035
5	1.567825	0.000021	0.000011	0.000031

Table 2: Specifications of b_0, b_1 for Data Set II

Order	Estimate of a	Estimate of b	b_0	b_1
4	1.813119	0.000161	0.000131	0.000191
5	1.461278	0.000173	0.000083	0.000263

Using the selected b_0, b_1 and $m_0(t)$, $m_1(t)$ we have calculated the decision rules given by Equations (14), (15), sequentially at each 't' of the data sets taking the strength (α, β) as (0.05,0.02). These are presented for the model in Table 3.

Table 3: SPRT Analysis for LPETM

Data Set	Order	T	N(t)	Acceptance Region (≤)	Rejection Region (≥)	Decision
1	4	1576	1	-2.52604	2.643752	REJECTED
		4149	2	-2.49925	2.799893	
		5827	3	-2.4831	2.897549	
	5	2610	1	-2.83529	2.988795	REJECTED
		4436	2	-2.83158	3.087522	
		8163	3	-2.82594	3.279789	
		11836	4	-2.8225	3.458517	
2	4	112	1	-7.84572	7.909789	REJECTED
		293.5	2	-7.90535	8.070809	
		473.5	3	-7.96383	8.227056	
		630.5	4	-8.01431	8.360666	
		793.5	5	-8.0662	8.496861	
		955.5	6	-8.11726	8.629777	
		1171.5	7	-8.18456	8.803376	
		1323.5	8	-8.23139	8.92316	
		1443.5	9	-8.26807	9.016392	
		1810.5	10	-8.37862	9.294562	
	5	112.5	1	-2.54988	2.600663	REJECTED
		358.5	2	-2.5437	2.701942	
		615.5	3	-2.53815	2.80384	

VI. Conclusion

In this paper we have monitored two failure live data sets using SPRT. We are greatly succeeded in applying SPRT analysis over order statistic approach. We have observed that through order statistic approach we can have an early decision about acceptance/ rejection of the software system being tested.

References

[1] J. D. Musa, K. Okumoto,"A logarithmic Poisson execution time model for software reliability measurement", in Proc. 7th Int. Conf. Software Eng., 1984, pp. 230-238.
[2] Kazuhira Okumoto,"A Statistical Method for Software Quality Control", In IEEE Transactions on Software Engg., Vol. SE-11, No. 12, Dec 1985.
[3] M.Xie, T.N. Goh, P.Ranjan,"Some Effective control chart procedures for reliability monitoring", Elsevier, Reliability Engineering & System Safety 77(2002), pp. 143-150.
[4] N. Boffoli, G. Bruno, D. Caivano, G. Mastelloni, "Statistical Process Control for oftware: a Systematic Approach", ESEM'08, October 9–10, 2008, Kaiserslautern, Germany, pp. 327-329, Copyright 2008 ACM 978-1-59593-971-5/08/10.
[5] Shewhart, Walter A.,"The Economic Control of Quality of Manufactured Product", D. Van Nostrand Company, New York, 1931, reprinted by ASQC Quality Press, Milwaukee, Wisconsin, 1980.
[6] In Jae Myung,"Tutorial on maximum likelihood estimation", Journal of Mathematical Psychology 47 (2003) 90–100.
[7] Hoang Pham,"A text book on, Software Reliability".
[8] Qin Zhou, Yunzhao Luo, Zhaojun Wang, "A control chart based on likelihood ratio test for detecting patterned mean and variance shifts", Computational Statistics and Data Analysis 54 (2010), pp. 1634-1645.
[9] Sunantha Teyarachakul, Suresh Chand, Jen Tang, "Estimating the limits for statistical process control charts:A direct method improving upon the bootstrap", European Journal of Operational Research 178 (2007), pp. 472–481.

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[10] R. R. L. Kantam, M. Ch. Priya,"Time Control Charts Using Order Statistics", Interstat.statjournals.net,2011.
[11] D. Haritha, R. Satya Prasad,"A Statistical Process Control to Monitor the Software Quality", IJCA Vol. 62, No. 11, January 2013. pp. 39-43.

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