Matrix Factorization of Image for Multimedia Mining

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Abstract

Matrix factorization techniques have been frequently applied in information retrieval, computer vision and pattern recognition. Among them, Non-negative Matrix Factorization (NMF) has received considerable attention due to its psychological and physiological interpretation of naturally occurring data whose representation may be parts-based in the human brain. On the other hand, from the geometric perspective, the data is usually sampled from a low dimensional manifold embedded in a high dimensional ambient space. One hopes then to find a compact representation which uncovers the hidden semantics and simultaneously respects the intrinsic geometric structure. This Paper presents novel algorithm, called Graph Regularized Non-negative Matrix Factorization of image for multimedia Mining (GNMFMM), In GNMFMM, an affinity graph is constructed to encode the image for multimedia, and seek a matrix factorization which respects the graph structure.

Keywords

Image, Matrix Factorization, Non-Negitive Matrixfactorization, Preprocessing

I. Introduction

The techniques for matrix factorization have become popular in recent years for data representation. In many problems in information retrieval, computer vision and pattern recognition, the input data matrix is of very high dimension. then to find two or more lower dimensional matrices whose product provides a good approximation to the original one. respects both hidden topics as well as geometric structure. In order to discover the hidden topics, matrix factorization techniques have been frequently applied For example, the canonical algorithm Latent Semantic Indexing applies Singular Value Decomposition (SVD) to decompose the original data matrix X into a product of three matrices, that is, X = USVT. U and V are orthogonal matrices and S is a diagonal matrix. Non-negative Matrix Factorization have been proposed and achieved great success due to its theoretical interpretation and practical performance. Previous studies have shown there is psychological and physiological evidence for parts-based representation in human brain. The non-negative constraints in NMF lead to a parts-based representation because it allows only additive, not subtractive, combinations. NMF has been shown to be superior to SVD in face recognition and document clustering. The major disadvantage of NMF is that it fails to consider the intrinsic geometric structure in the data. This paper, aim to discover the hidden topics and the intrinsic geometric structure simultaneously. propose a novel algorithm called Locality Preserving Non-negative Matrix Factorization (LPNMF) for this purpose. For two data points, we use KL-divergence to evaluate their similarity on the hidden topics. A nearest neighbor graph is constructed to model the local manifold structure. If two points are sufficiently close on the manifold, then we expect that they have similar representations on the hidden topics. Thus, the optimal maps are obtained such that the feature values on hidden topics are restricted to be non-negative and vary smoothly along the geodesics of the data manifold. We also propose an efficient method to solve the optimization problem. It is important to note that this work is fundamentally based on our previous work GNMF.

The rest of the paper is organized as follows: Section II, Related Work Section III, Problem description. Section IV, Frame Work Model. Experimental results are reported in Section V. Section VI, concludes the paper.

II. Related Work

Non-negative Matrix Factorization (NMF) is a matrix factorization algorithm that focuses on the analysis of data matrices whose elements are nonnegative. Given a data matrix $X = [x_1,…,x_n] \in R_{M \times N}$, each column of X is a sample vector. NMF aims to find two non-negative matrices $U = [uik] \in R^{M \times K}$ and $V = [v_{jk}] \in R^{N \times K}$ whose product can well approximate the original matrix X.

$X \approx UV^T$

There are two commonly used cost functions that quantify the quality of the approximation. The first one is the square of the Euclidean distance between two matrices (the square of the Frobenius norm of two matrices difference)

$$O_1 = \|X - UV^T\|^2 = \sum_{i,j} \left(x_{ij} - \sum_{k=1}^{K} u_{ik}v_{jk} \right)^2 \qquad \text{Eq.(1)}$$

The second one is the "divergence" between two matrices

$$O_2 = D(X\|UV^T) = \sum_{i,j} \left(x_{ij} \log \frac{x_{ij}}{y_{ij}} - x_{ij} + y_{ij} \right) \qquad \text{Eq. (2)}$$

where $Y = [y_{ij}] = UVT$. This cost function is referred to as "divergence" of X from Y instead of "distance" between X and Y because it is not symmetric. In other words, $D(X\|Y) \neq D(Y\|X)$. It reduces to the Kullback-Leibler divergence, or relative entropy, when $\sum_{ij} x_{ij} = \sum_{ij} y_{ij} = 1$ so that X and Y can be regarded as normalized probability distributions. We will refer O_1 as F-norm formulation and O_2 as divergence formulation in the rest of the paper. Although the objective function O_1 and O_2 are convex in U only or V only, they are not convex in both variables together. Therefore it is unrealistic to expect an algorithm to find the global minimum of O_1 (or, O_2). Lee & Seung presented two iterative update algorithms. The algorithm minimizing the objective function O_1 is as follows:

$$u_{ik} \leftarrow u_{ik} \frac{(XV)_{ik}}{(UV^TV)_{ik}}, \qquad v_{jk} \leftarrow v_{jk} \frac{(X^TU)_{jk}}{(VU^TU)_{jk}} \qquad \text{Eq. (3)}$$

The algorithm minimizing the objective function O_2 in

$$u_{ik} \leftarrow u_{ik} \frac{\sum_j (x_{ij}v_{jk} / \sum_k u_{ik}v_{jk})}{\sum_j v_{jk}}$$

$$v_{jk} \leftarrow v_{jk} \frac{\sum_i (x_{ij}u_{ik} / \sum_k u_{ik}v_{jk})}{\sum_i u_{ik}} \qquad \text{Eq. (4)}$$

It is proved that the above two algorithms will find local minima of the objective functions O1 and O2

III. Problem Description

By using the non-negative constraints, NMF can learn a parts-based representation. However, NMF performs this learning in the Euclidean space. It fails to discover the intrinsic geometrical and discriminating structure of the data space, which is essential to the real-world applications. In this section, we introduce our Graph

regularized Non-negative Matrix Factorization (GNMF) algorithm which avoids this limitation by incorporating a geometrically based regularizer.

A. NMF with Manifold Regularization

Recall that NMF tries to find a set of basis vectors that can be used to best approximate the data. One might further hope that the basis vectors can respect the intrinsic Riemannian structure, rather than ambient Euclidean structure. A natural assumption here could be that if two data points x_j, x_l are close in the intrinsic geometry of the data distribution, then z_j and z_l, the representations of this two points with respect to the new basis, are also close to each other. This assumption is usually referred to as local invariance assumption \which plays an essential role in the development of various kinds of algorithms including dimensionality reduction algorithms and semi-supervised learning algorithms. Recent studies in spectral graph theory and manifold learning theory have demonstrated that the local geometric structure can be effectively modeled through

a nearest neighbor graph on a scatter of data points. Consider a graph with N vertices where each vertex corresponds to a data point. For each data point x_j, we find its p nearest neighbors and put edges between x_j and its neighbors. There are many choices to define the weight matrix W on the graph. Given a data matrix X = [x_{ij}] $\in R^{M \times N}$, Our GNMF aims to find two non-negative matrices U = [u_{ik}] $\in R^{M \times K}$ and V = [v_{jk}] $\in R^{N \times K}$. Similar to NMF, we can also use two "distance" measures here. If the Euclidean distance is used, GNMF minimizes the objective function as follows:

$$\mathcal{O}_1 = \|X - UV^T\|^2 + \lambda \operatorname{Tr}(V^T L V).$$
Eq. (5)

If the divergence is used, GNMF minimizes

$$\mathcal{O}_2 = \sum_{i=1}^{M}\sum_{j=1}^{N}\left(x_{ij}\log\frac{x_{ij}}{\sum_{k=1}^{K}u_{ik}v_{jk}} - x_{ij} + \sum_{k=1}^{K}u_{ik}v_{jk}\right)$$
$$+ \frac{\lambda}{2}\sum_{j=1}^{N}\sum_{l=1}^{N}\sum_{k=1}^{K}\left(v_{jk}\log\frac{v_{jk}}{v_{lk}} + v_{lk}\log\frac{v_{lk}}{v_{jk}}\right)W_{jl}$$
Eq. (6)

B. Updating Rules Minimizing Eq.(5)

The objective function O_1 and O_2 of GNMF Eq.(5) and Eq. (6) are not convex in both U and V together. Therefore it is unrealistic to expect an algorithm to find the global minima. In the following, we introduce two iterative algorithms which can achieve local minima. We first discuss how to minimize the objective function O_1, which can be rewritten as

$$\mathcal{O}_1 = \operatorname{Tr}\left((X - UV^T)(X - UV^T)^T\right) + \lambda\operatorname{Tr}(V^T L V)$$
$$= \operatorname{Tr}(XX^T) - 2\operatorname{Tr}(XVU^T) + \operatorname{Tr}(UV^T V U^T)$$
$$+ \lambda\operatorname{Tr}(V^T L V)$$
Eq. (7)

$$\mathcal{L} = \operatorname{Tr}(XX^T) - 2\operatorname{Tr}(XVU^T) + \operatorname{Tr}(UV^T V U^T)$$
$$+ \lambda\operatorname{Tr}(V^T L V) + \operatorname{Tr}(\Psi U^T) + \operatorname{Tr}(\Phi V^T)$$
Eq. (8)

The partial derivatives of L with respect to U and V are:

$$\frac{\partial \mathcal{L}}{\partial U} = -2XV + 2UV^T V + \Psi$$
$$\frac{\partial \mathcal{L}}{\partial V} = -2X^T U + 2VU^T U + 2\lambda L V + \Phi$$
Eq. (9)

Using the KKT conditions

$$\psi_{ik}u_{ik} = 0 \text{ and } \phi_{jk}v_{jk} = 0,$$

We get the following equations for u_{ik} and v_{jk}:

$$-(XV)_{ik}u_{ik} + (UV^T V)_{ik}u_{ik} = 0$$

$$-(X^T U)_{jk}v_{jk} + (VU^T U)_{jk}v_{jk} + \lambda(LV)_{jk}v_{jk} = 0$$
Eq. (10)

These equations lead to the following updating rules:

$$u_{ik} \leftarrow u_{ik}\frac{(XV)_{ik}}{(UV^T V)_{ik}}$$
Eq. (11)

$$v_{jk} \leftarrow v_{jk}\frac{(X^T U + \lambda W V)_{jk}}{(VU^T U + \lambda D V)_{jk}}$$
Eq. (12)

IV. Frame Work Model

For the objective function of NMF, it is easy to check that if U and V are the solution, then, UD, VD^{-1} will also form a solution for any positive diagonal matrix D. To eliminate this uncertainty, in practice people will further require that the Euclidean length of each column vector in matrix U (or V) is. The matrix V (or U) will be adjusted accordingly so that UVT does not change. Our GNMF also adopts this strategy. After the multiplicative updating procedure converges, we set the Euclidean length of each column vector in matrix U to 1 and adjust the matrix V so that UVT does not change.

Now it is clear that the multiplicative updating rules in Eq. (11) and Eq. (12) are special cases of gradient descent with an automatic step parameter selection. The advantage of multiplicative updating rules is the guarantee of non-negativity of U and V.
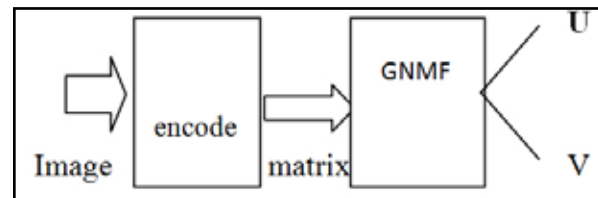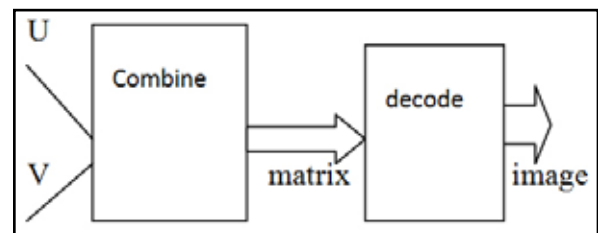

Fig. 1: Encoding

Fig. 2: Decoding

V. Results

Fig. 3(a): Image Before Factorization

Fig. 3(b): Image After Factorization

NMF is very powerful for clustering, especially in the document clustering and image clustering tasks. It can achieve similar or better performance than most of the state-of-the-art clustering algorithms, including the popular spectral clustering methods. Assume that a document corpus is comprised of K clusters each of which corresponds to a coherent topic. To accurately cluster the given document corpus, it is ideal to project the documents into a K-dimensional semantic space in which each axis corresponds to a particular topic. In this semantic space, each document can be represented as a linear combination of the K topics. Because it is more natural to consider each document as an additive rather than a subtractive mixture of the underlying topics, the combination coefficients should all take non-negative values.

These values can be used to decide the cluster membership. In appearance-based visual analysis, an image may be also associated with some hidden parts. For example, a face image can be thought of as a combination of nose, mouth, eyes, etc. It is also reasonable to require the combination coefficients to be non-negative. This is the main motivation of applying NMF on document and image clustering. In this section, we also evaluate our GNMF algorithm on document and image clustering problems.

VI. Conclusion

This paper presented a novel method for matrix factorization, called Graph regularized Non-negative Matrix Factorization (GNMF). GNMF models the data space as a submanifold embedded in the ambient space and performs the non-negative matrix factorization on this manifold. As a result, GNMF can have more discriminating power than the ordinary NMF approach which only considers the Euclidean structure of the data. Experimental results on document and image clustering show that GNMF provides a better representation in the sense of semantic structure.

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