Some Results on Derivatives of Generalized Modified Baskakov Type Operators (C)

[1]Dr. R. P. Pathak, [2]Shiv Kumar Sahoo
[1,2]Dept. of Mathematics, National Institute of Technology, Raipur, CG, India

Abstract
Recently Deo N.et.al. (Appl. Maths. Compt., 201(2008), 604-612.) Introduced a new Bernstein type special operators. Motivated by Deo N.et.al., in this paper we introduce generalization of modified Baskakov type positive linear operators (4) which is generator of positive linear operators (6) and (7) .We shall study some approximation results on it.

Keywords
Positive linear operators, Bernstein type special operators, Baskakov type positive linear operators.

I. Introduction

Baskakov [6] introduced the sequences of positive linear operators $\{L_n\},\ (n\epsilon N)$

$L_n : C[0,\infty) \rightarrow C[0,R]\ (R>0)$

which are defined by

$$(L_nf)(x) = \sum_{k=0}^{\infty} \frac{(-x)^k}{k!}\,\phi_n^{(k)}(x)\,f\left(\frac{k}{n}\right) \tag{1}$$

and are generated by sequence of functions $\{\phi_n\},\ (n\epsilon N)$ possesses the following five properties:

1. ϕ_n is analytic on the interval [0,R] where R>0 including end point.
2. $\phi_n(0)=1$.
3. ϕ_n is completely monotone i.e.

$$(-1)^k\phi_n^{(k)}(x) \geq 0 \qquad if\ \ k = 0,1,2,\ldots$$

4. There exist positive integer m(n) not depending on k such that

$$-\phi_n^{(k)}(x) = n\phi_{m(n)}^{(k-1)}(x)[1+\alpha_{k,n}(x)]$$

where $\alpha_{k,n}(x)$ converges to zero uniformly in k as $n\rightarrow\infty$.

5. $\lim_{n\to\infty}\frac{n}{m(n)} = 1.$

Deo N.et.al. [1] introduced a new Bernstein type special operators $\{V_n f\}$ defined as,

$$(V_nf)(x) = \sum_{k=0}^{n} p_{n,k}(x) f\left(\frac{k}{n}\right) \tag{2}$$

$$where\ \ p_{n,k}(x) = \left(1+\frac{1}{n}\right)^n \binom{n}{k} x^k\ ;$$

$$for\ \ 0 \leq x \leq \frac{n}{n+1}$$

Again Deo N.et.al. [1] gave the integral modification of the operators (2) which are defined as,

$$(L_nf)(x) = n\left(1+\frac{1}{n}\right)^2 \sum_{k=0}^{n} p_{n,k}(x)\int_0^{\frac{n}{n+1}} p_{n.k}(t)\,f(t)dt \tag{3}$$

and prove some approximation results on the operators (3).

Motivated by Deo N.et.al.[1] we studied a sequence of positive linear operators $\{M_n f\}$ which are defined as,

$$(M_nf)(x) = n\left(1+\frac{1}{n}\right)^2 \sum_{k=0}^{\infty}\frac{(-x)^k}{k!}\,\phi_n^{(k)}(x)\int_0^{\frac{n}{n+1}} p_{n.k}(t)\,f(t)dt$$

where

$$p_{n,k}(t) = \left(1+\frac{1}{n}\right)^n \binom{n}{k} t^k \left(\frac{n}{n+1}-t\right)^{n-k}$$

$$p>0 \quad and \quad for\ \ t\in\left[0,\frac{n}{n+1}\right] \ .$$

and are generated by sequence of functions $\{\phi_n\}$,$(n\epsilon N)$ possesses the above mentioned five properties. We shall study some approximation results on the operators (4).

The following are some special cases of the operators (4) :

A. $\phi_n(x) = e^{-nx}$

B. $\phi_n(x) = (1-x)^n$

C. $\phi_n(x) = (1+x)^{-n}$.

In this paper, we shall study some results on the positive linear operator (4) for special case $\phi_n(x)=(1+x)^{-n}$.

II. Main Results

In order to complete our study first we show that satisfies the above five properties (a) to (e).

1. Let $\phi_n(x)=(1+x)^{-n}$

Now successive differentiate the $\phi_n(x)$ k times w.r.t., we get

$$\phi_n'(x) = -n(1+x)^{-n-1}$$

$$\phi_n''(x) = (-1)^2n(n+1)(1+x)^{-n-2}$$

$$\phi_n'''(x) = (-1)^3n(n+1)(n+2)(1+x)^{-n-3}$$

$$\ldots\ldots\ldots\ldots\ldots$$

$$\phi_n^{(k)}(x) = (-1)^k\,n(n+1)(n+2)\ldots(n+k-1)\,(1+x)^{-n-k}$$

$$\phi_n^{(k)}(x) = (-1)^k\frac{n(n+1)(n+2)\ldots(n+k-1)\ (n-1)!}{(n-1)!}(1+x)^{-(n+k)}$$

$$\phi_n^{(k)}(x) = (-1)^k\frac{(n+k-1)!}{(n-1)!}(1+x)^{-(n+k)}$$

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Therefore ϕ_n (x) is differentiable in the interval $\left[0, \frac{n}{n+1}\right]$.

2. $\phi_n(0) = (1+o)^{-n} = 1^{-n} = 1.$

3. $If \ \ \phi_n(x) = (1+x)^{-n}$

$$\Rightarrow \phi_n^{(k)}(x) = (-1)^k \frac{(n+k-1)!}{(n-1)!}(1+x)^{-(n+k)}$$

$, k \le n$

$$\Rightarrow (-1)^k \phi_n^{(k)}(x)$$

$$= (-1)^k(-1)^k \frac{(n+k-1)!}{(n-1)!}(1+x)^{-(n+k)}$$

$$= (-1)^{2k} \frac{(n+k-1)!}{(n-1)!}(1+x)^{-(n+k)} \ge 0.$$

4. There exist positive integer m(n) not depending on such that

$$-\phi_n^{(k)}(x) = n\phi_{m(n)}^{(k-1)}(x)\left[1+\lim_{n\to\infty}\alpha_{k,n}(x)\right]$$

where, $\lim_{n\to\infty}\alpha_{k,n}(x) = 0$.

Since $\phi_n^{(k)}(x) = (-1)^k \frac{(n+k-1)!}{(n-1)!}(1+x)^{-(n+k)}$

$$\Rightarrow \phi_{n+1}^{(k-1)}(x) = (-1)^{k-1}\frac{(n+k-1)!}{n!}(1+x)^{-(n+k)}$$

Therefore

$$(-1)\ \phi_n^{(k)}(x)$$

$$= (-1)(-1)^k \frac{(n+k-1)!}{(n-1)!}(1+x)^{-(n+k)}$$

$$-\phi_n^{(k)}(x) = (-1)^{k-1} \frac{n\ \ (n+k-1)!}{n!}(1+x)^{-(n+k)}$$

$$= n\phi_{n+1}^{(k-1)}(x)$$

$$= n\phi_{n+1}^{(k-1)}(x)\ \left[1+\lim_{n\to\infty}\alpha_{k,n}(x)\right].$$

5. $\lim_{n\to\infty}\frac{n}{m(n)} \ \ = \lim_{n\to\infty}\frac{n}{n+c} = 1.$

Here we took c=1.

This completes the proof.

If we put the value of $\phi_n^{(k)}(x)$ in the operator (4), we get new modified positive linear operator.

$$(M_n f)(x)$$

$$= n\left(1+\frac{1}{n}\right)^2 \sum_{k=0}^{\infty} \frac{(-x)^k}{k!}\ (-1)^k \frac{(n+k-1)!}{(n-1)!}\ ..$$

$$..(1+x)^{-(n+k)}\int_0^{\frac{n}{n+1}} p_{n.k}(t)\, f(t)dt$$

$$= n\left(1+\frac{1}{n}\right)^2 \sum_{k=0}^{\infty} \frac{(-1)^{2k}\,x^k}{k!}\ \frac{(n+k-1)!}{(n-1)!}\ ..$$

$$...(1+x)^{-(n+k)}\int_0^{\frac{n}{n+1}} p_{n.k}(t)\, f(t)dt$$

$$= n\left(1+\frac{1}{n}\right)^2 \sum_{k=0}^{\infty} \frac{(n+k-1)!}{k!\,(n-1)!}\ \frac{x^k}{(1+x)^{(n+k)}}\ ..$$

$$...\int_0^{\frac{n}{n+1}} p_{n.k}(t)\, f(t)dt$$

$$= n\left(1+\frac{1}{n}\right)^2\ \sum_{k=0}^{\infty}\binom{n+k-1}{k}\frac{x^k}{(1+x)^{(n+k)}}$$

$$\int_0^{\frac{n}{n+1}} p_{n.k}(t)\, f(t)dt \quad .$$

Which is modified Baskakov type positive linear operators.

Now we are modifying operator (6) for c>0 as follows:

$$(A_n f)(x)$$

$$= n\left(1+\frac{1}{n}\right)^2 \sum_{k=0}^{\infty}\binom{k+\frac{n}{c}-1}{k}\frac{(cx)^k}{(1+cx)^{\frac{n}{c}+k}}$$

$$\int_0^{\frac{n}{n+1}} p_{n.k}(t)\, f(t)dt$$

$$\text{where } \ p_{n,k}(t) = \left(1+\frac{1}{n}\right)^n \binom{n}{k} t^k \left(\frac{n}{n+1}-t\right)^{n-k};$$

$$c>0 \ \ and \qquad for\ \ t \in \left[0, \frac{n}{n+1}\right] \quad .$$

Which is modified Baskakov type positive linear operators given by S. P. Singh et. al. (Proceeding of The Mathematical Society BHU Vol. 24 (2008) 1-9).

These operators have different approx properties, analysis is different. We will discuss them elsewhere.

III. Conclusion

In this paper, we conclude that our new modified Baskakov type operator (4) is generalized positive linear operator of modified Baskakov type operators (6) and (7). These operators (6) have different approx properties; analysis is different which is open question to study some approximation results for new researchers.

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