

# A Design Approach to Rudder Controller

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## Abstract

The design of a rudder controller for autonomous ship is a challenging task. Designer has to consider various parameters like ship dynamics, sea dynamics as well as the type of control mechanism. Researchers have come up with various ship models, sea models and different control algorithms in order to improve the performance of track keeping. This paper discusses the most widely used Autonomous ship models like Nomoto, Pierson-Markowitz (P-M) model and predictive controllers in detail with MATLAB simulated results.

## Keywords

PID, Nomoto, Wave Spectrums, Controllers.

## I. Introduction

To achieve the efficient ship track keeping/course keeping of ship and to generate accurate heading angles, one should have a robust controller which takes the following parameters into consideration ,such as sea disturbances, ship hydrodynamics and both internal and external noises. Design of robust rudder controller depends mainly on mathematical model of ship, sea disturbance as well type of controller algorithm.

There are two fundamental ways to approach the mathematical model. First one is classical modeling, which involves the analytical approach. In this, model is constructed based on physical knowledge of the system and further because of the computational and other practical requirements; the order of the model must be reduced for synthesis, analysis and implementation of the control systems.

Another way to create mathematical model is an experimental approach, in which the experiments are performed on the system and model is then fitted to the recorded data by assigning suitable numerical values to its parameters. This procedure is called system identification.

There are many classical ship modeling techniques [1] such as Nomoto model, Norrbinn model, Bech Model etc. All these modeling techniques have different design flow and methodology. The Nomoto Model is one of the simple linear models that are used for modeling of ship dynamics which supports all 6 degree of freedom (DOF) and is suitable for smaller rudder angles.

There are several techniques [2-4], available for wave modeling, such as Bretschneider spectrum, Pierson-Markowitz (P-M) spectrum, Modified Pierson-Markowitz and JONSWAP spectrum. P-M spectrum is simple and widely used since it supports fully developed sea elevation, which takes wide range of wave frequencies at all possible wind speeds into consideration.

There are three main controllers used in autonomous ship design, namely PID controllers, Adaptive Controllers and Advanced Predictive controllers. Since PID controller [5] cannot adapt (constant gain parameters) itself for time varying conditions it is not suitable for ship navigation. The Adaptive controller [6] is a non linear controllers which are efficient than PID controllers.

The parameters of adaptive controllers can modify themselves in accordance with the variation in sea dynamics, but these controllers are complex and computationally intensive and also it fails to predict the future control outputs .

Predictive controller's [7-9], offers both adaptability and prediction capability. Thus it is best suited for ship navigation. Predictive controller is based on various predictive models [10], like Auto Regressive Moving Average (ARMA), Auto Regressive Moving Average with exogenous input (ARMAX), and Controlled Auto Regressive Integrated Moving Average (CARIMA) etc. In this paper an attempt is made to explain Nomoto ship model, Pierson-Markowitz sea model and CARIMA predictive controller design steps with MATLAB simulation results.

The paper is organized as follows: section II, discusses the mathematical modeling of Nomoto ship from the fundamentals. Section-III, will provide the analysis of Pierson-Markowitz sea wave model and Section-IV, provide the detailed design of the controller and finally Section-V, provides and results and conclusion of the work.

## II. Modeling of Ship

There are many ship modeling techniques [1, 11], such as Nomoto model, Norrbinn model, Bech's Model etc. All these modeling techniques have different design flow and various levels of computational complexities. The Nomoto Model is simple, Linear and most widely used ship model for autopilot designs, This model provide better performance for smaller rudder angle turnings and best suitable for big ships since it supports all 6 DOF . The Bech's and Norrbinn models are non linear models that are derived from the basic Nomoto model. These models are used for larger rudder angle. This section discusses the steps involved in developing the relationship between change in heading angle with respect to the rudder angle.

The design flow of the Nomoto ship modeling starts from the analysis of basic motion/force equations to final transfer function of the ship model.

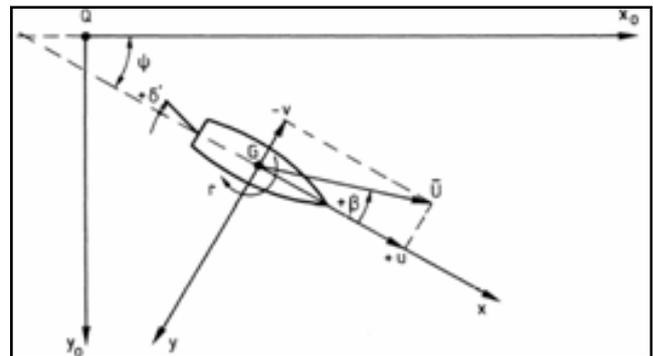


Fig. 1: Ship Coordinate System

To analyze ship dynamics it is convenient to define a coordinate system as indicated in fig. 1. The ship's center of gravity G is

chosen as the origin and the axes of symmetry are chosen as x-, y- and z-axes. The motion equations [1, 12] in the x, y and z directions along with rotations around these axes are as follows

$$m(v + u_0 r) = Y_V V + Y_{\dot{V}} \dot{V} + Y_{\phi} \phi + Y_P P + Y_{\dot{P}} \dot{P} + Y_r r + Y_{\dot{r}} \dot{r} + Y_{\delta} \delta \quad (1)$$

$$I_X \ddot{\phi} = K_P P + K_{\dot{P}} \dot{P} - mg \overline{GM} \phi + K_V V + K_{\dot{V}} \dot{V} + K_r r + K_{\dot{r}} \dot{r} + K_{\delta} \delta \quad (2)$$

$$I_Z \ddot{\psi} = N_r r + N_{\dot{r}} \dot{r} + N_{\phi} \phi + N_P P + N_{\dot{P}} \dot{P} + N_V V + N_{\dot{V}} \dot{V} + N_{\delta} \delta \quad (3)$$

The ship Maneuvering parameters are listed in Table 1.

Table 1: Ship Maneuvering Parameters

	Heading Angle
$\Phi$	Roll Angle
$r = d\psi / dt$	Yaw rate
$\dot{p}$	Roll Rate
$\delta$	Rudder angle
$u = dx / dt$	surge velocity
$v = dy / dt$	sway velocity
X, Y	Forces in the x- or y- direction
M, N, K	Moment components along X, Y, Z axis
m	Mass of Ship
$I_z$	Moment of Inertia about z-axis
$I_x$	Moment of Inertia about x-axis
$Y_v$	Sway force Y to Sway speed V
$K_r$	Z-axis Moment to Yaw rate

These equations are sufficient to represent six degrees of freedom and to balance both internal and external forces.

For surface ships, when motions in the horizontal plane are considered, only Surge (X), Sway(y), yaw (z) and roll(y) are considered, reducing the degrees of motion to four.

**A. Derivation of Transfer Function**

The transfer function of ship model [12] can be derived from the above given force or motion equations. The basic Transfer function in terms of Laplace coefficients is given below

$$\frac{\dot{\psi}}{\delta} = \frac{a_1(b_1 c_4 + b_4 c_3) + a_2(b_4 c_2 - b_2 c_4) + a_4(b_1 c_2 + b_2 c_3)}{a_1(b_1 c_1 - b_3 c_3) - a_2(b_2 c_1 + b_3 c_2) - a_3(b_1 c_2 + b_2 c_3)}$$

The coefficients of above Transfer function can be obtained by taking Laplace transform of (1)-(3) motion equations as follows,

$$a1V = a2\Phi + a3r + a4 \delta \quad (4)$$

$$b1 \Phi = b2V + b3r + b4 \delta \quad (5)$$

$$c1r = c2V + c3\Phi + c4 \delta \quad (6)$$

where a1, a2, a3, a4, b1, b2, b3, b4, c1, c2, c3, c4 are expressed as follows,

$$a1 = (m - Y_V)S - Y_V \quad (7)$$

$$a2 = Y_{\dot{P}} S^2 + Y_P S + Y_{\phi} \quad (8)$$

$$a3 = Y_r S + Y_r + m u_0 \quad (9)$$

$$a4 = Y_{\delta} \quad (10)$$

$$b1 = (I_X - K_{\dot{P}})S^2 - K_P S + mg \overline{GM} \quad (11)$$

$$b2 = K_{\dot{V}} S + K_V \quad (12)$$

$$b3 = K_r S + K_r \quad (13)$$

$$b4 = K_{\delta} \quad (14)$$

$$c1 = (I_Z - N_r)S - N_r \quad (15)$$

$$c2 = N_{\dot{V}} S + N_V \quad (16)$$

$$c3 = N_{\dot{P}} S^2 + N_P S + N_{\phi} \quad (17)$$

$$c4 = N_{\delta} \quad (18)$$

The above Laplace coefficients are substituted in the basic transfer function and after simplification, the transfer function is given by

$$\frac{\dot{\psi}}{\delta} = \frac{K(1 + T_3 S)(S^2 + 2\eta w_o S + w_o^2)}{(1 + T_1 S)(1 + T_2 S)(S^2 + 2w_n S + w_n^2)} \quad (20)$$

$T_1, T_2,$  and  $T_3$  are the time constants and K is yaw gain constant which depends on physical parameters of ship.

**B. Simplification of Nomoto's Transfer Function**

The Eq. (20) is the fourth order Nomotos ship model. It consists of 2 zeros and 3 poles. The quadratic factors are due to the coupling effect from the roll mode on the yaw rate. The zero  $(1 + T_3 S)$  and the pole  $(1 + T_2 S)$  are due to the coupling effect from the sway mode on the yaw dynamics. If the roll mode is neglected, Eq. (20) can be further reduced to the following form

$$\frac{\dot{\psi}}{\delta} = \frac{K(1 + T_3 S)}{(1 + T_1 S)(1 + T_2 S)} \quad (21)$$

Eq. (21) is known as the second order Nomoto model, where K is the static yaw rate gain, and  $T_1, T_2$  and  $T_3$  are time constants. The zero term  $(1 + T_3 S)$  and the high frequency pole term  $(1 + T_2 S)$  are due to the coupling effect from the sway mode. In practice, because the pole term  $(1 + T_2 S)$  and the zero term  $(1 + T_3 S)$  in Eq. (21) nearly cancel each other, a further simplification on Eq. (21) can be done to get the first order Nomoto model

$$\frac{\dot{\psi}}{\delta} = \frac{K}{(1 + TS)} \quad (22)$$

Where  $T = T_1 + T_2 - T_3$ , T is called as effective yaw rate time constant.

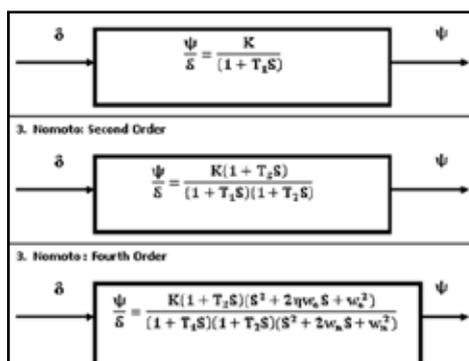


Fig. 2: Nomoto 1st, 2nd and 4th Order Ship Models

The 4th order Nomotos model in Eq. (21) supports all ship dynamics but this model is computationally intensive. The first order Nomoto model defined by Eq. (22) is simple and widely employed in the ship steering autopilot design and it supports only yaw dynamics (heading angle). For high speed ships, roll dynamics need to be considered; hence first order model may not be suitable for autopilot design. The second order Nomotos model is relatively simple and supports for both yaw and roll dynamics, so this model is best suitable for autopilot design. The ship steering autopilots are designed for heading angle control, so there is a necessity to develop transfer function relating the heading angle  $\Psi$  to the rudder angle  $\delta$  for designing the controller.

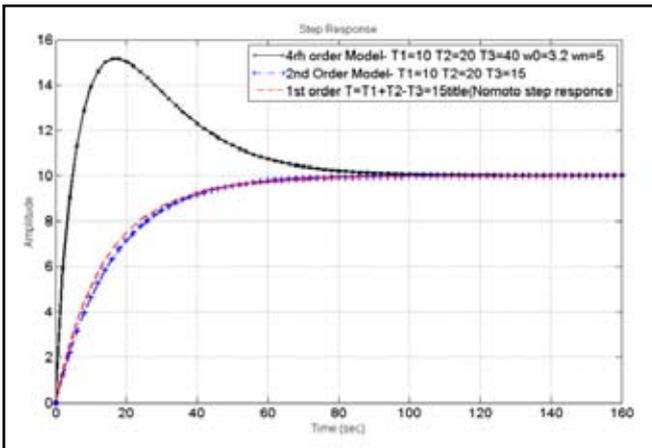


Fig. 3: Step Response of 1<sup>st</sup> 2<sup>nd</sup> and 4<sup>th</sup> Order Nomoto’s Model by MATLAB

The Nomoto’s model simulation results are as shown in fig. 3, the step response of 4th order nomoto model has overshoot of 51%, so it leads to error in turning calculation. The settling time for 1st, 2nd and 4th order models for the step response in the figure 3 are 131, 128, 122 sec respectively. The 1st order model response is acceptable if roll effect is neglected. The 2nd order model is almost same as 1st order but takes roll effect into consideration so 2nd order model provides optimal solution.

### III. Modelling of Sea Wave Disturbances

The undesirable motion of a ship in a seaway is induced by the action of environmental disturbances: waves, wind, and current. However, ocean waves are the dominant environmental disturbances. The sea waves are a random phenomenon, meaning that the attributes of waves cannot be easily captured for computational purpose since the irregular sea never repeats its pattern from one interval to another. At the same time understanding the dynamics of sea states are crucial to predict the movement of ship. Waves may have quite different origins and characteristics [10]. However, for the design of an autopilot it suffices to consider only waves generated by the wind. In general, the pattern of the waves is rather complex. The resulting pattern is a summation of waves with different amplitudes, phases and frequencies and with various directions of propagation. The description of waves can be simplified by disregarding the different directions of propagation: only unidirectional waves will be described. The stochastic nature of the waves can be taken into account by describing the waves by means of a frequency spectrum. Such a spectrum can be obtained from measured data of wave motions by applying fast Fourier transformation. As per literature survey wave spectrum can be analytically described as a function of the wind speed or as a function of the significant wave height and the average period.

### A. Standard Wave Spectrums

There are various methods available to model the sea waves [10-11], the most generally referred standard wave spectrums are :

- Bretschneider spectrum
- Pearson-Markowitz Spectrum
- Modified Pearson-Markowitz Spectrum
- JONSWAP Wave Spectrum

In this section a detailed analysis of Pearson-Markowitz Spectrum and Modified Pearson-Markowitz Spectrum is carried out with MATLAB simulation results. The general mathematical equation of wave spectrum equation is given below

$$S_{\epsilon}(w) = \frac{A}{w^5} \exp\left(\frac{-B}{w^4}\right) [m^2 s] \quad (23)$$

Where  $w$  is Wave frequency and the parameters  $A, B$  are related to the modal frequency  $w_0$  and the Spectral moments ( $m^0$ ). The  $A$  and  $B$  parameters varies for all the spectrums. The modal frequency is given by,

$$w_0 = \left(\frac{4B}{5}\right)^{\frac{1}{4}} \quad (24)$$

The spectral moments is obtained as

$$m_{\zeta}^0 = \frac{A}{4B} \quad m_{\zeta}^1 = 0.3 \frac{A}{B^{3/4}} \quad m_{\zeta}^2 = \sqrt{\frac{\pi A^2}{16B}} \quad (25)$$

The above moments represents rising and falling sea waves, as well as fully developed seas with no swell.

#### 1. Pierson-Markowitz Spectrum(P-M)

The Pierson-Markowitz [12] was developed to forecast Storm waves at a single point in fully developed seas with no swell This relates the parameters  $A$  and  $B$  to the average wind speed at 19.5m above the sea surface as

$$A = 8.1 \times 10^{-3} g^2 \quad B = \frac{0.74g}{V_{19.5}} \quad (26)$$

$$S_{\epsilon}(w) = \frac{8.1 \times 10^{-3} g^2}{w^5} \exp\left(\frac{-0.74}{V_{19.5} w^4}\right) [m^2 s] \quad (27)$$

The P-M wave spectrum in terms of the wave height and wave frequency is as given below

$$S(\omega) = \frac{8.1 \times 10^{-3} g^2}{\omega^5} \exp\left(-\frac{3.11}{H_{1/3}^2 \omega^4}\right) \quad (28)$$

This wave spectrum is narrow band. The energy is mainly concentrated at a certain band. The equal frequency division method is used to select limited harmonic waves to simulate the wave. The bandwidth of spectrum is can be calculated using the formulas 29 and 30.

$$\omega_L = \left(-\frac{3.11}{H_{1/3}^2 \ln \mu}\right)^{1/4} \quad (29)$$

$$\omega_H = \left(-\frac{3.11}{H_{1/3}^2 \ln(1-\mu)}\right)^{1/4} \quad (30)$$

$$\Delta\omega = \frac{\omega_H - \omega_L}{M} \quad (M= 50\sim 100) \quad (31)$$

#### 2. Modified Pierson-Markowitz Spectrum(P-M)

This is same as P-M spectrum, but,  $A$  and  $B$  parameters are represented in terms of significant wave height ( $H_{1/3}$ ) and different wave period ( $T_0, T_1, T_2$  etc) statistics as given below

$$A = \frac{487H_{1/3}^2}{T_0} = \frac{173H_{1/3}^2}{T_1} = \frac{123H_{1/3}^2}{T_z}, \tag{32}$$

$$B = \frac{1949}{T_0^4} = \frac{691}{T_1^4} = \frac{495}{T_z^4}$$

Table 1: Sea states with wave heights

Sea state	H <sub>1/3</sub> lower limit	H <sub>1/3</sub> upper limit	Seaway description
0	0	0	Calm (glassy)
1	0	0.1	Calm (rippled)
2	0.1	0.5	Smooth
3	0.5	1.25	Slight
4	1.25	2.5	Moderate
5	2.5	4	Rough
6	4	6	Very rough
7	6	9	High
8	9	14	Very high
9	14	>14	Phenomenal

Table 1 shows the sea states commonly used to describe the seaway in marine applications and fig. 4, shows the behavior of P-M spectrum at different sea wave heights.

The fig. 5, shows the simulation of comparison of modified P-M, Bretschneider spectrum and JONSWAP wave spectrums. The JONSWAP spectrum has got highest peak (supports high sea states) but it is a narrowband spectrum covers short range of wave frequencies (partially developed). The Bretschneider spectrum covers wide range of frequency but its peak is very less, so useful for smaller waves or sea states. The P-M spectrum has moderate peak and wider frequency range. Thus P-M spectrum supports for fully developed wave elevations.

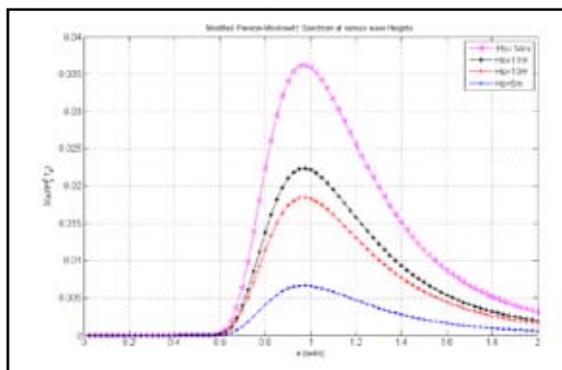


Fig. 4: P-M Spectrum at Various Sea Wave Heights by MATLAB

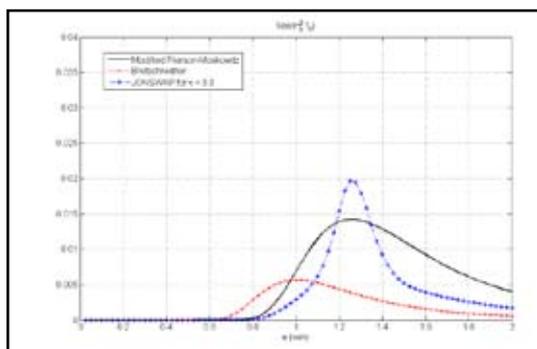


Fig. 5: Various Wave Spectrum Comparison Simulation by MATLAB

### C. Encounter Frequency

The wave frequency experienced by the hull of a ship is called the encounter frequency, and it is denoted by  $\omega_e$ . The encounter frequency plays an important role in determining the intensity of wave's effect on ship hull. When the ship is at zero speed, the frequency at which the waves excite the ship same as wave frequency  $\omega$ . However, when the ship moves, the frequency observed from the ship differs from the wave frequency. Thus encounter frequency at ship hull need to be determined. If the ship moves forwards with an average speed  $U$  with wave encounter is represented as shown in fig. 6. it shows that the relative speed at which the waves overtake the ship is  $c - U \cos(\chi)$ . Then, we can express the encounter frequency as

$$\omega_e = \frac{2\pi}{T_e} = \frac{2\pi}{\lambda} [c - U \cos(\chi)] \tag{33}$$

Where  $\chi$ - encounter angle

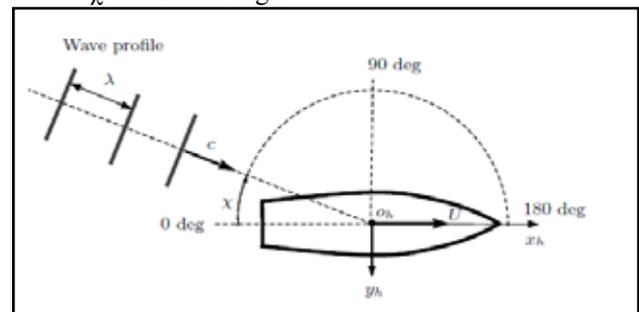


Fig. 6: Encounter Frequency Analysis

### D. Simulation of Wave Elevation

For surface ships, ocean waves are the dominant environmental disturbances. So the analysis of sea surface elevation at various sea states is required. The sea surface elevation can be considered as periodic wave system having a continuous energy spectrum. The wave elevation can be represented as  $\zeta(t)$  on the time interval  $[0, T]$ . The mathematical model of wave elevation is given below

$$\zeta(t) = \sum_{n=1}^N \bar{\zeta}_n \cos(\omega_n t + \epsilon_n) \tag{34}$$

$N$  – Number of harmonic waves sufficiently large,

$\epsilon_n$  – Phases differences of waves, random variable which is distributed evenly between 0 and  $2\pi$ .

$\bar{\zeta}_n$  - Maximum amplitude of each harmonic wave in the random wave.

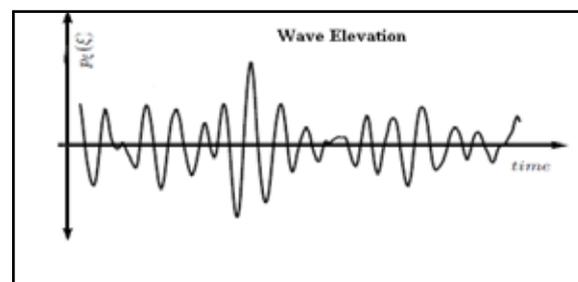


Fig. 7: Wave Elevation Analysis

The Maximum amplitude  $\bar{\zeta}_n$  in terms of wave spectrum  $S$  and frequency band  $\Delta\omega$  is given by

$$\bar{\zeta}_n = \sqrt{2S(\omega)\Delta\omega} \tag{35}$$

$\Delta\omega$ - Bandwidth or range of the wave spectrum

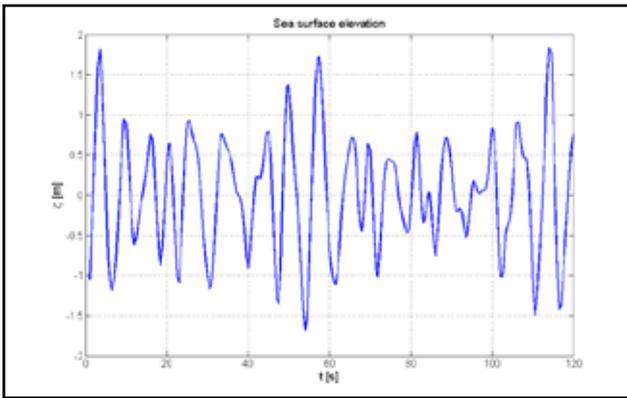


Fig. 8: Sea Wave Elevation at Moderate Sea State Simulation by MATLAB

To simulate the wave disturbance for a ship, we have to identify the type of the wave spectrum we have considered and the range of that spectrum, wave height or sea state. The figure 8 shows the wave simulation for P-M spectrum and for the moderate sea state (Wave height 2-6m)

**IV. Controller Design**

This section provides predictive control design by Controlled auto Regressive Integrated Moving average (CARIMA) model.

The general CARIMA model is given by

$$\Delta A(z^{-1})y(k) = B(z^{-1})\Delta u(k) + n_c(k) \tag{36}$$

A, B -polynomials of  $z^{-1}$

Y (k) – Plant (computed) output

u(k)- input to the plant

$n_c(k)$ - unmeasured Noise or disturbances.

CARIMA model uses the numerator and denominator of ship transfer function to acquire prediction of the future heading angles by assuming present and past inputs and past outputs.

The design steps of the controller are given below

Step 1: Derivation of Nomoto’s ship transfer function

From the section II, the derivation of 2nd order nomoto’s ship model is given by

$$\frac{\psi}{\delta} = \frac{K(1 + T_3S)}{(1 + T_1S)(1 + T_2S)} \tag{37}$$

Here K is Yaw gain constant and  $T_1, T_2, T_3$  are time constants of the ship model.

Step 2: Discretization of the Nomoto’s ship T.F

The above transfer function is converted to discrete form is given below

$$\frac{\psi}{\delta} = \frac{b_1z^{-1} + b_2z^{-2}}{1 + a_1z^{-1} + a_2z^{-2}} \tag{38}$$

Step3: Extraction of A, B Polynomials for CARIMA model

A polynomial is extracted from the denominator, which acts as an auto regressive component. B polynomial is extracted from numerator of Eq. 37 acts as moving average component.

$$B(z^{-1}) = b_1z^{-1} + b_2z^{-2} \tag{39}$$

$$A(z^{-1}) = 1 + a_1z^{-1} + a_2z^{-2} \tag{40}$$

Step 4: Transferring Nomoto’s into CARIMA Model

Put A and B polynomials into CARIMA model in Eq.35 and after simplification; the output of the model is given by

$$y(k) = (1 - a_1)y(k - 1) + (a_1 - a_2)y(k - 2) + a_2y(k - 3) + b_1u(k - 1) + b_2u(k - 2) \tag{41}$$

It was necessary to compute three step ahead predictions in straightforward way by establishing of lower predictions to higher predictions. The model order defines that computation of one step ahead prediction. It is based on three past values of the system output in case of a second order model. The three step ahead predictions are as follows

$$y(k + 1) = (1 - a_1)y(k) + (a_1 - a_2)y(k - 1) + a_2y(k - 2) + b_1u(k) + b_2u(k - 1) \tag{42}$$

$$y(k + 2) = (1 - a_1)y(k + 1) + (a_1 - a_2)y(k) + a_2y(k - 1) + b_1u(k + 1) + b_2u(k) \tag{43}$$

$$y(k + 3) = (1 - a_1)y(k + 2) + (a_1 - a_2)y(k + 1) + a_2y(k) + b_1u(k + 2) + b_2u(k + 1) \tag{44}$$

The Prediction equations can be represented in the form of matrix shown below [4]

$$\begin{bmatrix} y(k+1) \\ y(k+2) \\ y(k+3) \end{bmatrix} = \begin{bmatrix} g_1 & 0 \\ g_2 & g_1 \\ g_3 & g_2 \end{bmatrix} \begin{bmatrix} \Delta u(k) \\ \Delta u(k+1) \end{bmatrix} + \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \end{bmatrix} \begin{bmatrix} y(k) \\ y(k-1) \\ y(k-2) \\ \Delta u(k-1) \end{bmatrix} = \begin{bmatrix} b_1 & 0 \\ b_1(1-a_1)+b_2 & b_1 \\ (a_1-a_2)b_1+(1-a_1)^2b_1+(1-a_1)b_2 & b_1(1-a_1)+b_2 \end{bmatrix} \times \begin{bmatrix} \Delta u(k) \\ \Delta u(k+1) \end{bmatrix} + \begin{bmatrix} (1-a_1) & (a_1-a_2) \\ (1-a_1)^2+(a_1-a_2) & (1-a_1)(a_1-a_2)+a_2 \\ (1-a_1)^2+2(1-a_1)(a_1-a_2)+a_2 & (1-a_1)^2(a_1-a_2)+a_2(1-a_1)+(a_1-a_2)^2 \end{bmatrix} \times \begin{bmatrix} y(k) \\ y(k-1) \\ y(k-2) \\ \Delta u(k-1) \end{bmatrix} \tag{45}$$

It is possible to divide computation of the predictions to recursion of the free response and recursion of the matrix of the dynamics. Based on the three previous predictions it is repeatedly computed the next row of the free response matrix in the following way

$$\begin{aligned} p_{41} &= (1 - a_1)p_{31} + (a_1 - a_2)p_{21} + a_2p_{11} \\ p_{42} &= (1 - a_1)p_{32} + (a_1 - a_2)p_{22} + a_2p_{12} \\ p_{43} &= (1 - a_1)p_{33} + (a_1 - a_2)p_{23} + a_2p_{13} \\ p_{44} &= (1 - a_1)p_{34} + (a_1 - a_2)p_{24} + a_2p_{14} \end{aligned} \tag{46}$$

The first row of the matrix is omitted in the next step and further prediction is computed based on the three last rows including the one computed in the previous step. This procedure [13] is cyclically repeated. It is possible to compute an arbitrary number of rows of the matrix. The recursion of the dynamics matrix is similar.

The next element of the first column is repeatedly computed in the same way as in the previous case. If the control horizon is lower than the prediction horizon a number of columns in the matrix is reduced.

$$g_4 = (1 - a_1)g_3 + (a_1 - a_2)g_2 + a_2g_1 \quad (47)$$

This procedure is performed repeatedly until the prediction horizon is achieved.

**V. Results & Conclusion**

**A. Simulation using MATLAB**

The model introduced in section II is used to simulate the real dynamics of the ship. The ship data for Eq.(20) are obtained from [14]. The following transfer function of the ship is derived.

$$TF = \frac{-1.041e^{-17}s - 7.886e^{-9}}{s^2 + 0.00208s - 0.0002279} \quad (48)$$

Model Predictive controller provides an accurate medium to simulate the controller behaviour under specific operating conditions. The Controller-plant pair is modelled using Simulink and simulated under various conditions comprising of varying orders of wave-spectrum. The second order plant takes rudder command as the input manipulated variable while the heading angle and the rate of change of heading angle is considered as measured and unmeasured outputs respectively. The predictive controller is designed with the following parameters.

Table 2: MPC Controller Specifications

Control interval	1.0
Prediction horizon	30
Control horizon	5

The control interval is chosen such that the plant’s open-loop settling time is approximately 20–30 sampling periods. Fig. 9, shows rudder angle generated by Model predictive controller for a 30° initial input and disturbances along with previous heading angle feedback.

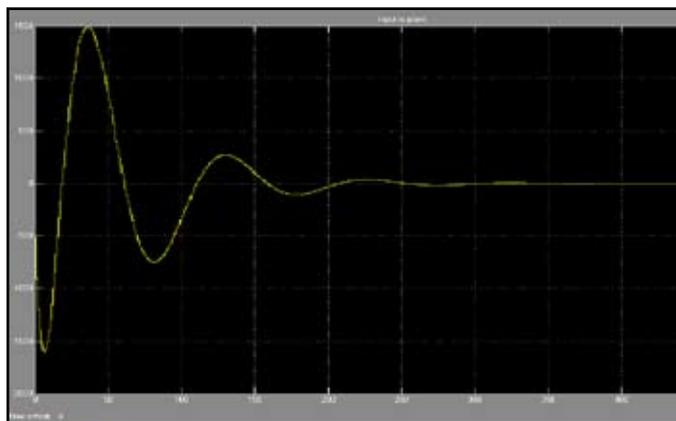


Fig. 9: Input Rudder Angle to the Plant/Output of the Controller

The generated rudder angle by the Model predictive controller is given as input to ship. The output response of the ship or heading angle for a 30° initial rudder angle input is as shown in fig. 10. It shows that the output heading angle is following the input rudder angle with slight variations due to wave disturbances and system

delay (Inference is drawn from fig. 9 and fig. 10).

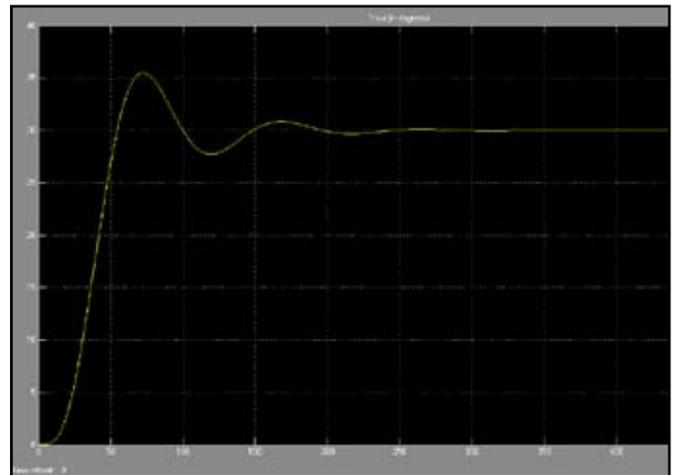


Fig. 10: Output Heading Angle of the Ship (deg)

Fig. 11, shows the navigation plot which compares the real trajectory with the obtained trajectory in the presence

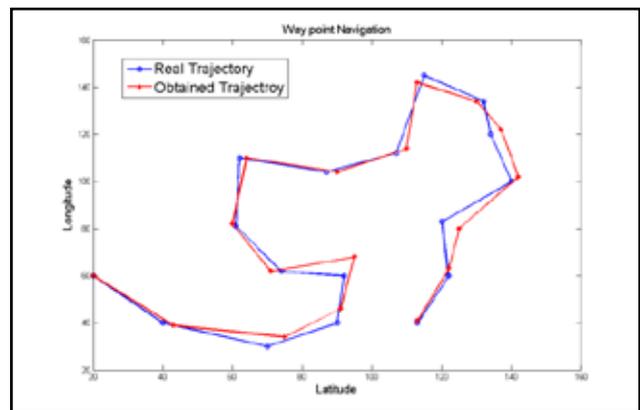


Fig. 11: Comparison of Real Trajectory and Obtained trajectory

of the various sea disturbances and other internal errors. It is observed that precise navigation is obtained from the use of predictive controller. It can be concluded from this paper that the design of rudder controller depends on the ship model, wave model and the type of controller.

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