# A Novel Approach of Linear Parallel Interference Cancellation in Uplink OFDMA Systems with Carrier Frequency Offsets

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**Abstract**
In uplink orthogonal frequency division multiple access (OFDMA) systems, multi-user interference (MUI) occur due to different carrier frequency offsets (CFO) of different users at the receiver. Here, we present a multistage linear parallel interference cancellation (LPIC) approach to mitigate the effect of this MUI in uplink OFDMA. The proposed scheme first performs CFO compensation (in time-domain) followed by K DFT operations (where K is the number of users) and multistage LPIC on these DFT outputs. We scale the MUI estimates by weights before cancellation and optimize these weights by maximizing the average signal-to-interference ratio (SIR) at the output of the different stages of the LPIC. We derive closed-form expressions for these optimum weights. The proposed LPIC scheme is shown to effectively cancel the MUI caused by the other user CFOs in uplink OFDMA. We show that our approach performs better than the existing technique by comparing the bit error rates of the existing system and our proposed system.

**Keywords**
Linear Parallel Interference Cancellation (LPIC), Weighted Linear Parallel Interference Cancellation (WLPIC), Carrier Frequency Offset (CFO), Multiple-user Interference (MUI).

## I. Introduction

The principles of orthogonal frequency division multiplexing (OFDM) modulation have been around for several decades. However in recent years, this technology has quickly moved out of the academia world into the real world of modern communication systems. OFDM is a multi-carrier modulation scheme that encodes data on to a radio frequency (RF) signal. Unlike conventional single-carrier modulation schemes–such as AM/FM (amplitude or frequency modulation)–that send only one signal at a time using one radio frequency, OFDM sends multiple high-speed signals concurrently on specially computed, orthogonal carrier frequencies. The result is much more efficient use of bandwidth as well as robust communications during noise and other interferences. OFDM [1] in its primary form is considered as a digital modulation technique, and not a multi-user channel access technique, since it is utilized for transferring one bit stream over one communication channel using one sequence of OFDM symbols. However, OFDM can be combined with multiple access using time, frequency or coding separation of the users. In Orthogonal Frequency Division Multiple Access (OFDMA), frequency-division multiple access is achieved by assigning different OFDM sub-channels to different users. OFDMA supports differentiated quality-of-service by assigning different number of sub-carriers to different users in a similar fashion as in CDMA [12], and thus complex packet scheduling or media access control schemes can be avoided. Orthogonal Frequency-Division Multiple Access (OFDMA) is a multi-user version of the popular Orthogonal frequency-division multiplexing (OFDM) digital modulation scheme. Multiple access is achieved in OFDMA by assigning subsets of subcarriers to individual users.

This allows simultaneous low data rate transmission from several users. Orthogonal Frequency Division Multiple Access (OFDMA) [9]-[10] assigns a subset of subcarriers to individual users and the transmission is simultaneous, as shown in Fig.1. Each OFDMA user transmits symbols using subcarriers that remain orthogonal to those of other users. More than one subcarrier can be assigned to support high rate applications. Simultaneous transmission from several users results in better spectral efficiency.

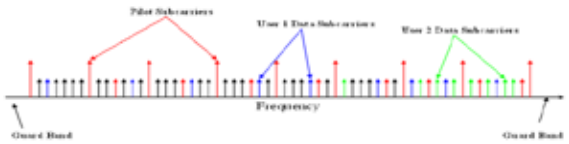


Fig. 1. OFDMA- Multiple Access

## II. OFDMA

OFDMA can be seen as an alternative to combining OFDM with time division multiple access (TDMA) or time-domain statistical multiplexing, i.e. packet mode communication. Low-data-rate users can send continuously with low transmission power instead of using a "pulsed" high-power carrier. Constant delay, and shorter delay, can be achieved. OFDMA is considered as highly suitable for broadband wireless networks, due to advantages including scalability and MIMO-friendliness, and ability to take advantage of channel frequency selectivity.

Multicarrier modulation schemes, as in OFDMA, eliminate or minimize inter-symbol interference (ISI) by making the symbol time large enough so that the channel-induced delays—delay spread being a good measure of this in wireless channels—are an insignificant fraction of the symbol duration. Therefore, in high-data-rate systems in which the symbol duration is small, being inversely proportional to the data rate, splitting the data stream into many parallel streams increases the symbol duration of each stream such that the delay spread is only a small fraction of the symbol duration.

Multiple-access schemes are orthogonal when an ideal receiver can completely reject arbitrarily strong unwanted signals using different basis functions than the desired signal.

CFO refers to the difference in carrier frequency at transmitter and receiver. The transmitter would transmit at the nominal carrier frequency. At the Receiver the unmodulated frequency is required for reception; and is usually generated in the Receiver with crystals or so. However, it is not physically possible to make the Receiver frequency exactly match the Transmitter frequency. This offset is termed CFO.

A time interval left vacant (i.e., during which no data is sent) on a transmission channel that can be used for synchronization and/or compensating for a signal distortion is known as guard interval. To eliminate ISI almost completely, a guard time is introduced for each symbol.

In the uplink, each user's signal may be distorted by an independent carrier frequency offset (CFO)[2],[6],[8]-[10], which impairs the orthogonality of the subcarrier signals and, if not properly compensated, results multiuser interference (MUI) along with significant performance degradations. The presence of a carrier frequency offset (CFO) introduces not only ICI (inter-carrier interference) and ISI but also multiple access interference (MAI) between users in the OFDMA uplink.
Multiple-access schemes are said to be orthogonal when an ideal receiver can completely reject arbitrarily strong unwanted signals using different basis functions than the desired signal.

## III. Idealized System Model

An uplink OFDMA system is considered with K users, where each user communicates with a base station through an independent multipath channel as shown in Fig.2.

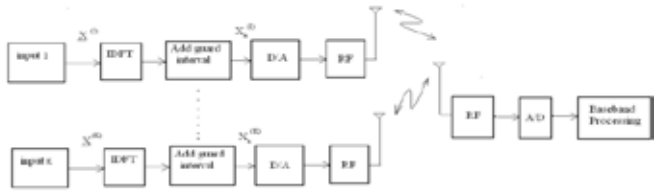

Fig. 2. Uplink OFDMA System Model

It is assumed that there are N subcarriers in each OFDM symbol and one subcarrier can be allocated to only one user. The information symbol for the ith user on the kth subcarrier is denoted by $X_k^{(i)}$, where $S_i$ is the set of subcarriers assigned to user i and $\mathrm{E}\left[\left|X_k^{(i)}\right|^2\right]=1$.

Then, $\cup_{i=1}^{k} S_i = \{0,1,2,\ldots\ldots,N-1\}$ and $S_i \cap S_j = \phi,$ for i ≠ j
The length of the guard interval added is Ng samples and is assumed to be longer than the maximum channel delay spread. After IDFT processing and guard interval insertion at the transmitter, the time-domain sequence of the ith user $x_n^{(i)}$, is given by

$$x_n^{(i)} = \frac{1}{N}\sum_{k\in s_i} X_k^{(i)} e^{\frac{j2\pi nk}{N}}, -N_g \le n \le N-1 \quad (1)$$

The ith user's signal at the receiver input, after passing through the channel, is given by

$$s_n^{(i)} = x_n^{(i)} * h_n^{(i)} \quad (2)$$

where * denotes linear convolution and $h_n^{(i)}$ is the ith user's channel impulse response. It is assumed that $h_n^{(i)}$ is non-zero only for n = 0, . . ., L − 1, where L is the maximum channel delay spread, and that all users' channels are statistically independent.

An assumption is made that $h_n^{(i)}$ are i.i.d. complex Gaussian with zero mean and $E\left[\left(h_{n,I}^{(i)}\right)^2\right] = E\left[\left(h_{n,Q}^{(i)}\right)^2\right] = 1/2L$, where $h_{n,I}^{(i)}$ and $h_{n,Q}^{(i)}$ are the real and imaginary parts of $h_n^{(i)}$. The channel coefficient in frequency-domain $H_k^{(i)}$ is given by

$$H_k^{(i)} = \sum_{n=0}^{L-1} h_n^{(i)} e^{\frac{-j2\pi nk}{N}}, \quad (3)$$

and $E\left[\left|H_k^{(i)}\right|^2\right] = 1$

The received baseband signal after coarse carrier frequency tracking (leaving some residual carrier frequency offset) is given by

$$r_n = \sum_{i=1}^{K} s_n^{(i)} e^{\frac{j2\pi nk}{N}} + z_n, -N_g \le n \le N-1 \quad (4)$$

where $\varepsilon_i$, i = 1,…, K denotes ith user's residual carrier frequency offset (CFO) normalized by the subcarrier spacing, and Zn is the AWGN with zero mean and variance σ2. We assume that all users are time synchronized and $\varepsilon_i$, i =1,. ,. , K is known at the receiver.

Fig. 3 shows the receiver baseband processing including
CFO compensation is in time-domain and
guard time removal,
K- DFT operations (one for each user), and
Linear parallel interference cancellation (LPIC) in multiple stages.

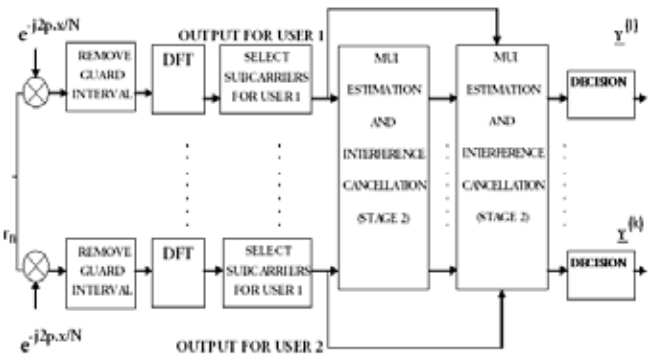


Fig. 3.Receiver baseband processing – CFO compensation and multistage interference cancellation

It should be noted that the CFO compensation is carried out in time-domain by multiplying $r_n$ with $e^{\frac{-j2\pi\varepsilon_i k}{N}}$, i = 1, ... ,K (this method of CFO compensation is referred to as the direct method).
The received signal after CFO compensation and guard time removal for the ith user is given by

$$y_n^{(i)} = r_n e^{\frac{-j2\pi nk}{N}}, \quad 0 \le n \le N-1 \quad (5)$$

which forms the input to the ith DFT block.
The output of the DFT block for the ith user on the kth subcarrier is then given by

$$Y_k^{(i)} = \mathrm{H}_k^{(i)} X_k^{(i)} + \sum_{\substack{l=1 \\ l\neq i}}^{K} \sum_{q\in s_l} \rho_{kq}^{(i),(l)} \mathrm{H}_q^{(l)} X_q^{(l)} + \mathrm{Z}_k^{(i)}, \quad (6)$$

$$\rho_{kq}^{(i),(l)} = \frac{\sin\pi(k-q+\delta_{li})}{N\sin\frac{\pi}{N}(k-q+\delta_{li})} e^{-j\left(1-\frac{1}{N}\right)\pi(k-q+\delta_{li})} \quad (7)$$

and $\delta_{li}$ is the difference between the ith user and lth user CFO values, given by

$$\delta_{li} = \varepsilon_l - \varepsilon_i \quad (8)$$

The channel coefficient $H_k^{(i)}$ $H_k^{(i)}$ is given by (3) and the noise component $Z_k^{(i)}$ is given by

$$Z_k^{(i)} = \sum_{n=0}^{N-1} \mathrm{Z}_n^{(i)} e^{\frac{-j2\pi n(k+\varepsilon_i)}{N}}, \quad (9)$$

It should be noted that the 2nd term in (6) represents the CFO-induced MUI present at the DFT output. In the case of single user detection (SUD), the DFT outputs, $Y_k^{(i)}$'s, can be directly used to make the symbol decision. Additional processing may be performed on $Y_k^{(i)}$'s in order to mitigate the effect of MUI. For example, multistage interference cancellation techniques can be employed to improve performance. In the next chapter, a multistage weighted linear parallel interference cancellation scheme which operates on the DFT outputs $Y_k^{(i)}$ is proposed.

## IV. Proposed WLPIC Scheme

It is noted that, for the desired user i, the 2nd term in (6) represents the CFO-induced MUI (i.e., interference from other users $l = 1, 2, \ldots, \mathrm{K}, l \neq \mathrm{i}$) present at the DFT output. Also, from Equations (6), (7) and (8), it can be noted that the amount of this MUI depends on $\delta_{li}$'s, the difference between desired user CFO ($\varepsilon_{i,}$) and other user CFOs ($\varepsilon_{l,}$). The aim here is to cancel this 2nd term in (6) using a multistage linear PIC approach, where an estimate of the MUI in a given stage is obtained using the soft values of previous stage outputs (without any non-linear operation, e.g., hard decision, on the previous stage outputs).
Further, it is known that the MUI estimates in an LPIC approach can become quite inaccurate under poor channel conditions (e.g., low SNR, high interference) to such an extent that it may be better not to do cancellation. Such situations can be alleviated by scaling the MUI estimates by weights (preferably by some optimum weights) before cancellation. Here, such a weighted LPIC (WLPIC) scheme for the uplink OFDMA is presented. The proposed multistage WLPIC scheme is shown in Fig. 3.

Let m denote the stage index. The DFT outputs, $Y_k^{(i)}$ in (6) are taken as the first stage (m = 1) outputs of the receiver, i.e. $Y_{k,(1)}^{(i)} = Y_k^{(i)}$. In the case of SUD, the symbol decisions are made directly from $Y_{k,(1)}^{(i)}$. LPIC is performed in the subsequent stages. In a given LPIC stage m, m > 1, an estimate of the MUI is made based on the soft values of the previous stage outputs. These MUI estimates are scaled by weights and subtracted (i.e., cancelled) from the DFT outputs, $Y_{k,(1)}^{(i)}$
As mentioned earlier, the 2nd term in (6) is seeked to cancel. Towards that end, the following operation is considered on the other user DFT outputs, $Y_{k,(1)}^{(i)}$
It is seeked to cancel the second term in DFT block output. Hence the following operation on other user DFT outputs $Y_{q,(1)}^{(l)}$ is considered,

$$\sum_{\substack{l=1 \\ l\neq i}}^{K} \sum_{q\in s_l} \rho_{kq}^{(i),(l)} Y_{q,(1)}^{(l)} \quad (10)$$

Using (6) in the above, it can be written

$$\sum_{\substack{l=1 \\ l\neq i}}^{K} \sum_{q\in s_l} \rho_{kq}^{(i),(l)} Y_{q,(1)}^{(l)} = \sum_{\substack{l=1 \\ l\neq i}}^{K} \sum_{q\in s_l} \rho_{kq}^{(i),(l)} \left( \mathrm{H}_q^{(l)} X_q^{(l)} + \sum_{\substack{j=1 \\ j\neq l}}^{K} \sum_{r\in s_i} \rho_{qr}^{(l),(j)} \mathrm{H}_r^{(j)} X_r^{(j)} + \mathrm{Z}_q^{(l)} \right)$$

$$= \sum_{\substack{l=1 \\ l\neq i}}^{K} \sum_{q\in s_l} \rho_{kq}^{(i),(l)} \mathrm{H}_q^{(l)} X_q^{(l)}$$

$$+ \sum_{\substack{l=1 \\ l\neq i}}^{K} \sum_{q\in s_l} \rho_{kq}^{(i),(l)} \sum_{\substack{j=1 \\ j\neq i}}^{K} \sum_{r\in s_j} \rho_{qr}^{(l),(j)} \mathrm{H}_r^{(j)} X_r^{(j)}$$

$$+ \sum_{\substack{l=1 \\ l\neq i}}^{K} \sum_{q\in s_l} \rho_{kq}^{(i),(l)} Z_q^{(l)} \quad (11)$$

It should be noted that the 1st term T1 in (11) is the same as the MUI term (i.e., the 2nd term) in (6), which we wanted to cancel. Hence, (10) can be viewed as an MUI estimate for the 2nd stage of the LPIC, which when cancelled (i.e., subtracted) from (6) will completely remove the MUI term (i.e., the 2nd term) in (6). In the process, additional terms T2 (interference) and T3 (noise), which were not there in the 1st stage output, get introduced in the 2nd stage output. The interference term T2 introduced in the 2nd stage can be cancelled in the 3rd stage using a similar MUI estimate obtained from $Y_{q,(2)}^{(l)}$, and so on. Accordingly, in the proposed WLPIC, the interference cancelled output of the ith user on the kth subcarrier in the mth stage,

$Y_{k,(m)}^{(i)}$, m > 1, can be written as

$$Y_{k,(m)}^{(i)} = Y_{k,(1)}^{(i)} - \omega_{k,(m)}^{(i)} \sum_{\substack{l=1 \\ l\neq i}}^{K} \sum_{q\in s_l} \rho_{kq}^{(i),(l)} Y_{q,(m-1)}^{(l)} \quad (12)$$

Where $Y_{k,(1)}^{(i)}$ is the 1st stage output given by (6), $\rho_{kq}^{(i),(l)}$ is given by (7), and $\omega_{k,(m)}^{(i)}$ is the weight with which the MUI estimate is scaled.

The WLPIC scheme with the weights on all subcarriers of all users to be unity $\omega_{k,(m)}^{(i)} = 1$ is called as conventional LPIC (CLPIC) scheme. In the CLPIC scheme, the operations needed for the choice of optimum weights and MUI scaling with these weights are avoided (because of unity weights). However, performance better than that of the CLPIC can be achieved by using optimum weights. A proposal to obtain the optimum weights for the mth stage, $\omega_{k,(m)}^{(i),opt}, i = 1, 2....., K, k \in S_i$, by maximizing the corres- ponding average SIR at the mth stage output is made.

In an uncoded system, the symbol decision for the ith user on the kth subcarrier at the output of the mth stage can be made based on the output $Y_{k,(m)}^{(i)}$. For example, the symbol decision at the mth stage output for the case of BPSK modulation can be obtained as

$$\hat{X}_{k,(m)}^{(i)} = \mathrm{sgn}\left(\mathrm{Re}\left(\overline{H_k^{(i)}} Y_{k,(m)}^{(i)}\right)\right) \quad (13)$$

In the above equation and henceforth, 'overline' is used to denote conjugate operation, i.e., $\overline{H_k^{(i)}}$ denotes the conjugate of $H_k^{(i)}$.

## V. Equations SIR Analysis and Optimum Weight Derivation

In this section, expressions for the average SIR at the output of the 2nd and 3rd stages of the proposed weighted LPIC scheme are derived. Also, the derived average SIR expressions are used to obtain closed-form expressions for the optimum weights $\omega_{k,(m)}^{(i),opt}$.

It is noted that the average output SIR on a given subcarrier will depend on several things including number of users, channel impulse response, number of subcarriers, CFO values, and type of subcarrier allocation. Here, two types of subcarrier allocation are considered, namely, i) block allocation and ii) interleaved allocation. In block allocation, a consecutive block of subcarriers is allotted to one user, the next block to another user, and so on. In interleaved allocation, the subcarriers of each user are uniformly interleaved with the subcarriers assigned to the other users.

### 5.1 Average SIR at the 2nd stage output

From (12), the weighted interference cancelled output of the 2nd stage (m = 2) for the ith user on the kth subcarrier is given by

$$Y_{k,(2)}^{(i)} = \mathrm{H}_k^{(i)} X_k^{(i)} \left(1 - \omega_{k,(2)}^{(i)} \sum_{\substack{l=1 \\ l\neq i}}^{K} \sum_{q\in s_l} \rho_{kq}^{(i),(l)} \rho_{qk}^{(l),(i)}\right) + I_2 + N_2 \quad (14)$$

Where

$$I_2 = \sum_{\substack{l=1 \\ l\neq i}}^{K} \sum_{\substack{p\in s_l \\ p\neq k}} H_p^{(l)} X_p^{(l)} \left(\rho_{kp}^{(i),(l)} \left(1 - \omega_{k,(2)}^{(i)}\right) - \omega_{k,(2)}^{(i)} \sum_{\substack{u=1 \\ u\neq i,l}}^{K} \sum_{u\in s_u} \rho_{kv}^{(i),(u)} \rho_{vk}^{(u),(l)}\right) \quad (15)$$

and

$$\mathrm{N}_2 = \mathrm{Z}_k^{(i)} - \omega_{k,(2)}^{(i)} \sum_{\substack{l=1 \\ l\neq i}}^{K} \sum_{q\in s_l} \rho_{kq}^{(i),(l)} Z_i^{(l)} \quad (16)$$

The terms I2 and N2 in (15) and (16) represent the interference and noise terms introduced due to imperfect cancellation in using the soft output values from the first stage. After coherent combining using $\overline{H_k^{(i)}}$, the final output is given by

$$\overline{H_k^{(i)}} Y_{k,(2)}^{(i)} = \left|\mathrm{H}_k^{(i)}\right|^2 X_k^{(i)} \left(1 - \omega_{k,(2)}^{(i)} \sum_{\substack{l=1 \\ l\neq i}}^{K} \sum_{q\in s_l} \rho_{kq}^{(i),(l)} \rho_{qk}^{(l),(i)}\right) + I_2^{'} + N_2^{'} \quad (17)$$

Where $I_2^{'} = \overline{\mathrm{H}_k^{(i)}} I_2$

$$N_2^{'} = \overline{\mathrm{H}_k^{(i)}} N_2$$

Considering the $\mathrm{H}_k^{(i)} \mathrm{H}_p^{(l)}$ factor in $I_2^{'}$, it is noted that the channel coefficients on subcarriers of different users i1 and i2, $\mathrm{H}_{k1}^{(i1)}$ and $\mathrm{H}_{k2}^{(i2)}$, k1 ∈ Si1, k2 ∈ Si2, i1 ≠ i2, are uncorrelated because all users' channels are assumed to be independent. However, from (1), it can be seen that the channel coefficients on different subcarriers of the same user i, $\mathrm{H}_{k1}^{(i1)}$ and $\mathrm{H}_{k2}^{(i2)}$, k1,k2 ∈ Si, are correlated. Also, this correlation depends on the subcarrier allocation. Handling the correlation between $\mathrm{H}_{k1}^{(i1)}$ and $\mathrm{H}_{k2}^{(i2)}$ in the SIR analysis is tedious. Therefore, to simplify the analysis, it is assumed that $\mathrm{H}_{k1}^{(i1)}$ and $\mathrm{H}_{k2}^{(i2)}$ are uncorrelated. Accordingly, the variance of $I_2^{'}$ and $\sigma_{I_2^{'}}^2$, can be obtained as

$$\sigma_{I_2^{'}}^2 = \left|\mathrm{H}_k^{(i)}\right|^2 \sigma_{I_2}^2 \quad (18)$$

Where

$$\sigma_{I_2}^2 = \sum_{\substack{l=1 \\ p\neq k}}^{K} \sum_{p\in s_l} \left| \rho_{kp}^{(i),(l)} \left(1 - \omega_{k,(2)}^{(i)}\right) - \omega_{k,(2)}^{(i)} \sum_{\substack{u=1 \\ u\neq i,l}}^{K} \sum_{u\in s_u} \rho_{kv}^{(i),(u)} \rho_{vk}^{(u),(l)} \right|^2 \quad (19)$$

And the variance of $N_2^{'}$ , $\sigma^2_{N_2^{'}}$ , can be obtained as

$$\sigma^2_{N_2^{'}} = \left|\mathrm{H}_k^{(i)}\right|^2 \sigma^2_{N_2}, \quad (20)$$

$$\sigma^2_{N_2} = \sigma^2 \left( N + 2N\left(\omega^{(i)}_{k,(2)}\right)^2 \sum_{\substack{l=1 \\ l\neq i}}^{K} \sum_{q\in s_l} \left|\rho^{(i),(l)}_{kq}\right|^2 + \right.$$

Where ,

$$\left.\left(\omega^{(i)}_{k,(2)}\right)^2 \sum_{\substack{l=1 \\ l\neq i}}^{K} \sum_{q\in s_l} \sum_{\substack{u=1 \\ u\neq i,l}}^{K} \sum_{u\in s_u} \rho^{(i),(l)}_{kq} \overline{\rho^{(i),(u)}_{kv}}\, \eta^{(i),(u)}_{qv} - \mathrm{Re}\left[ 2\omega^{(i)}_{k,(2)} \sum_{\substack{l=1 \\ l\neq i}}^{K} \sum_{q\in s_l} \rho^{(i),(l)}_{kq}\, \eta^{(l),(i)}_{qk} \right] \right)$$

(21)

and

$$\eta^{(l),(i)}_{qk} = E\left[ Z^{(l)}_q \overline{Z^{(i)}_k} \right] = \sum_{n=0}^{N-1} e^{\frac{-j2\pi n\left(\varepsilon_l - \varepsilon_i + k - q\right)}{N}} \quad (22)$$

The average SIR on the kth subcarrier of the ith user at the output of the 2nd stage, $SIR^{(i)}_{k,(2)}$ can then be obtained as

$$SIR^{(i)}_{k,(2)} = \frac{\left( 1 - \omega^{(i)}_{k,(2)} \sum_{\substack{l=1 \\ l\neq i}}^{K} \sum_{q\in s_l} \rho^{(i),(l)}_{kq} \rho^{(l),(i)}_{qk} \right)^2}{\sigma^2_{I_2^{'}} + \sigma^2_{N_2^{'}}} \quad (23)$$

## 5.2 Average SIR at the 2nd stage output

The soft values of the interference cancelled 2nd stage outputs of different users, $Y^l_q, l \neq i, q \in S_l$ , are used to reconstruct (estimate) the MUI on the kth subcarrier of the desired user i in the 3rd stage. These MUI estimates are then scaled by $\omega^{(i)}_{k,(3)}$ and cancelled. Accordingly, the weighted interference cancelled output of the 3rd stage for the ith user on the kth subcarrier $Y^{(i)}_{k,(3)}$ , is given by

$$Y^{(i)}_{k,(3)} = H^{(i)}_k X^{(i)}_k F + I_3 + N_3 \quad (24)$$

Where

$$F = 1 - w^{(i)}_{k,(3)} \sum_{\substack{l=1, \\ l\neq i}}^{K} \sum_{q\in s_l} \rho^{(i),(l)}_{kq} (\rho^{(l),(i)}_{qk} (1 - w^{(l)}_{q,(2)}) - w^{(l)}_{q,(2)} \sum_{\substack{u=1 \\ u\neq l,i}}^{K} \sum_{v\in s_u} \rho^{(l),(u)}_{qv} \rho^{(u),(i)}_{vk})$$

(25)

and

$$I_3 = \sum_{l=1}^{K} \sum_{\substack{q\in s_l \\ q\neq k}} H^{(l)}_q X^{(l)}_q \rho^{(i),(l)}_{kq} [(1 - w^{(i)}_{k,(3)} (1 - w^{(l)}_{q,(2)} \sum_{\substack{u=1 \\ u\neq l}}^{K} \sum_{v\in s_u} \rho^{(l),(u)}_{qv} \rho^{(u),(l)}_{vq}))$$

$$- w^{(i)}_{k,(3)} \sum_{\substack{u=1, \\ u\neq i,l}}^{K} \sum_{v\in s_u} \rho^{(l),(u)}_{kv} (\rho^{(u),(l)}_{vq} (1 - w^{(u)}_{v,(2)})$$

$$- w^{(u)}_{v,(2)} \sum_{\substack{n=1 \\ n\neq u,l}}^{K} \sum_{s\in s_n} \rho^{(u),(n)}_{vs} \rho^{(n),(l)}_{sq})] \quad (26)$$

And

$$N_3 = Z^{(i)}_k - \omega^{(i)}_{k,(3)} \sum_{\substack{l=1 \\ l\neq i}}^{K} \sum_{q\in s_l} \rho^{(i),(l)}_{kq} \left( Z^{(l)}_q - \omega^{(i)}_{k,(3)} \sum_{\substack{u=1 \\ u\neq l}}^{K} \sum_{r\in S_u} \rho^{(i),(u)}_{qv} Z^{(u)}_v \right) \quad (27)$$

After coherent combining using $H^{(i)}_k$ , the final output is given by

$$\overline{H^{(i)}_k} Y^{(i)}_{k,(3)} = \left|\mathrm{H}^{(i)}_k\right|^2 X^{(i)}_k + I_3^{'} + N_3^{'} \quad (28)$$

Where $I_3^{'} = \overline{\mathrm{H}^{(i)}_k} I_3$

and $N_3^{'} = \overline{\mathrm{H}^{(i)}_k} N_3$ .

Again, assuming $\mathrm{H}^{(i)}_{k1}$ and $\mathrm{H}^{(i)}_{k2}$ to be uncorrelated, the variance of $I_3^{'}$, $\sigma^2_{I_3^{'}}$ , and the variance of $N_3^{'}$, $\sigma^2_{N_3^{'}}$ ,respectively can be obtained, as

$$\sigma^2_{I_3^{'}} = \left|\mathrm{H}^{(i)}_k\right|^2 \sigma^2_{I_3} \quad (29)$$

Where

$$\sigma^2_{I3} = \sum_{l=1}^{K} \sum_{\substack{p\in s_l \\ p\neq k}} \left| \rho^{(i),(l)}_{kq} \left[ \left( 1 - \omega^{(i)}_{k,(3)} \left( 1 - \omega^{(i)}_{k,(2)} \sum_{\substack{u=1 \\ u\neq l}}^{K} \sum_{v\in s_u} \rho^{(l),(u)}_{qv} \rho^{(u),(l)}_{vq} \right) \right) \right.\right.$$

$$- \omega^{(i)}_{k,(3)} \sum_{\substack{u=1 \\ u\neq i,l}}^{K} \sum_{v\in s_u} \rho^{(l),(u)}_{kv} \left( \rho^{(u),(l)}_{vq} \left( 1 - \omega^{(u)}_{v,(2)} \right) \right.$$

$$\left.\left.\left. - \omega^{(u)}_{v,(2)} \sum_{\substack{u=1 \\ u\neq u,l}}^{K} \sum_{s\in s_u} \rho^{(u),(n)}_{vs} \rho^{(n),(l)}_{sq} \right) \right] \right|^2 \quad (30)$$

And

$$\sigma^2_{n_3^{'}} = \left|\mathrm{H}^{(i)}_k\right|^2 \sigma^2_{N_3} \quad (31)$$

Where $\sigma^2_{N_3}$ is given by

$$\sigma^2_{N_3} = \sigma^2 \left[ N \left( 1 + \omega^{(i)}_{k,(3)} \sum_{\substack{l=1 \\ l\neq i}}^{K} \sum_{q\in s_l} \rho^{(i),(l)}_{kq} \omega^{(l)}_{q,(2)} \rho^{(l),(i)}_{qk} \right)^2 \right.$$

$$+ N \left( \omega^{(i)}_{k,(3)} \right)^2 \sum_{\substack{u=1 \\ u\neq i}}^{K} \sum_{r\in S_u} \left| \left( -\rho^{(j),(u)}_{kr} + \sum_{\substack{l=1 \\ l\neq i,u}}^{K} \sum_{q\in s_l} \rho^{(i),(l)}_{kq} \rho^{(l),(i)}_{qr} \omega^{(l)}_{q,(2)} \right) \right|^2$$

$$\left. + N \left( \omega^{(i)}_{k,(3)} \right)^2 \sum_{r\in Si} \left| \sum_{\substack{l=1 \\ l\neq i}}^{K} \sum_{q\in s_l} \rho^{(i),(l)}_{kq} \rho^{(i),(i)}_{qr} \omega^{(l)}_{q,(2)} \right|^2 \right.$$

$$+\left(\omega_{k,(3)}^{(i)}\right)^2 \sum_{\substack{u=1\\u\neq i}}^{K} \sum_{r\in S_u} \sum_{\substack{c=1\\c\neq i,u}}^{K} \sum_{v\in S_c} \left(-\rho_{kr}^{(i),(u)} + \sum_{\substack{l=1\\l\neq i,u}}^{K} \sum_{q\in s_l} \rho_{kq}^{(i),(l)} \rho_{qr}^{(l),(i)} \omega_{q,(2)}^{(l)}\right)$$

$$\overline{\left(-\rho_{kr}^{(i),(c)} + \sum_{\substack{l=1\\l\neq i,c}}^{K} \sum_{q\in s_l} \rho_{kq}^{(i),(l)} \rho_{qv}^{(l),(i)} \omega_{q,(2)}^{(l)}\right)} \eta_{rv}^{(v),(c)}$$

$$+2\left(\omega_{k,(3)}^{(i)}\right)^2 \mathrm{Re}\left(\sum_{\substack{u=1\\u\neq i}}^{K} \sum_{r\in S_u} \sum_{v\in S_i} \left(-\rho_{kr}^{(i),(u)} + \sum_{\substack{l=1\\l\neq i,u}}^{K} \sum_{q\in s_l} \rho_{kq}^{(i),(l)} \rho_{qr}^{(l),(i)} \omega_{q,(2)}^{(l)}\right) \eta_{rv}^{(v),(i)} \overline{\left(\sum_{\substack{l=1\\l\neq i,c}}^{K} \sum_{q\in s_l} \rho_{kq}^{(i),(l)} \rho_{qr}^{(l),(i)} \omega_{q,(2)}^{(l)}\right)}\right) \quad (32)$$

The average SIR on the kth subcarrier of the ith user at the output of the 3rd stage, $SIR_{k,(3)}^{(i)}$, can then be obtained as

$$SIR_{k,(3)}^{(i)} = \frac{F^2}{\sigma_{I_3}^{2'} + \sigma_{N_3}^{2'}} \quad (33)$$

## VI. SIR Analysis

The channel model used is a one sample spaced, two-ray, equal-gain Rayleigh fading model. Perfect knowledge of $\in$ is assumed. The SIRs in the simulations are measured as follows.

For a given realization of the channel coefficients $H_k^{(i)}$, the total power in the received signal (i.e., power in the LHS in Equations (17) and (28) is computed,

using the knowledge of $H_k^{(i)}$'s and ρ's, the desired signal power is computed (i.e., power in the 1st term on RHS in Equations (17) and (28),

the difference between powers in i) and ii) gives the interference plus noise power, and

SIR is computed as the ratio of the powers in ii) and iii).

The average SIR is obtained over several realizations of the channel coefficients. The difference between the simulated SIR and the analytical SIR is that in the analysis to derive the interference variance it is assumed that $H_{k1}^{(i1)}$ and $H_{k2}^{(i2)}$, k1 $\in$ Si1, k2 $\in$ Si2, i1 $\neq$ i2, are uncorrelated, whereas this assumption is not there in the simulations.

The average SIR expressions in (23) and (33) can be maximized by differentiating and equating to zero to obtain optimum weights.

A. Closed-Form derivation of optimum weights

The average SIR expressions for the second and third stage outputs can be maximized to obtain optimum weights for scaling the interference estimate at the second and third stages.

1. Optimum weight of second stage, $\omega_{k,(2)}^{(i),opt}$ in closed- form

An expression for the optimum weights $\omega_{k,(2)}^{(i),opt}$ can be obtained by differentiating the second stage SIR equation with respect to $\omega_{k,(2)}^{(i)}$ and equating to zero.

Accordingly, we obtain the expression for $\omega_{k,(2)}^{(i),opt}$ as

$$\omega_{k,(2)}^{(i),opt} = -\frac{(\beta_1 + 2\beta_2\beta_3)}{(\beta_1\beta_2 + 2\beta_4)} \quad (34)$$

2. Optimum weight of third stage, $\omega_{k,(3)}^{(i),opt}$ in closed-form

Similarly, by differentiating the average SIR of the third stage with respect to $\omega_{k,(3)}^{(i)}$ and equating to zero, the expression for the optimum weights $\omega_{k,(3)}^{(i),opt}$, in closed-form is obtained, as

$$\omega_{k,(3)}^{(i),opt} = -\frac{(\gamma_1 + 2\gamma_2\gamma_3)}{(\gamma_1\gamma_2 + 2\gamma_4)} \quad (35)$$

## VII. Simulated Result Analysis

The following Fig. shows the Bit Error Rate (BER) comparison between the conventional linear parallel interference cancellation and the proposed weighted linear parallel interference cancellation at the second stage.

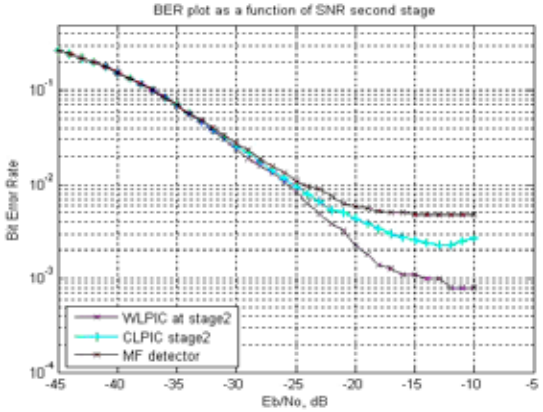


Fig. 4. Performance comparison (stage 2)

The following Fig. shows the Bit Error Rate (BER) comparison between the conventional linear parallel interference cancellation and the proposed weighted linear parallel interference cancellation at the third stage.

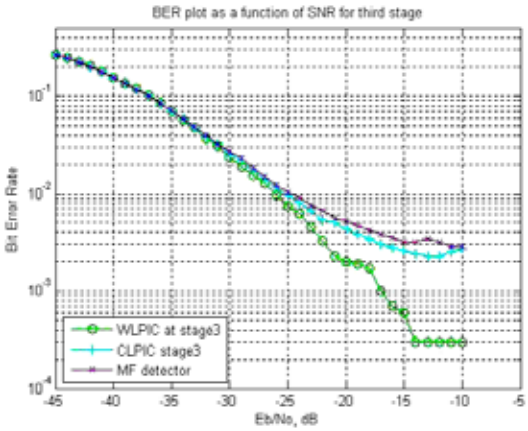


Fig. 5. Performance comparison (stage 3)

The following Fig. shows the Bit Error Rate (BER) comparison between the conventional linear parallel interference cancellation and the proposed weighted linear parallel interference cancellation at the second stage and the third stages.

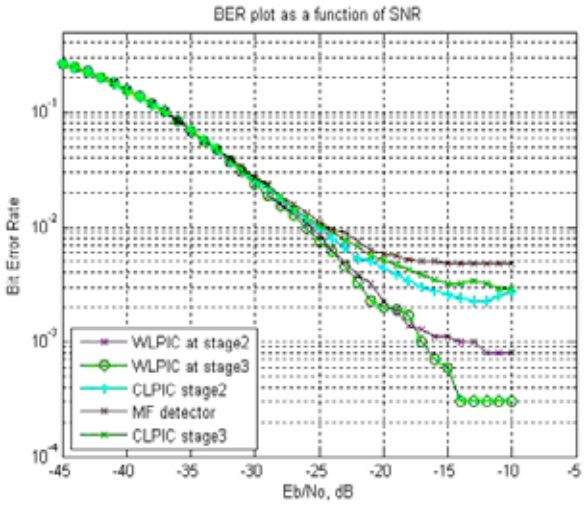


Fig. 6. Overall performance comparison

The system parameters are summarized in Table 1.

Table 1: List of system parameters

| Sl. NO | System Parameters | Values |
|---|---|---|
| 1 | Number of subcarriers (N) | 32 |
| 2 | Number of users (K) | 4 |
| 3 | Difference between $i^{th}$ and $l^{th}$ users' CFO values $[\in_1, \in_2, \in_3, \in_4]$ | [-0.1, 0.3, 0.25, -0.15] |
| 4 | SNR in dB | 25 dB |

Table 2: Eb/No Vs BER for Stage 2

| Sl. No | $E_b/N_o$ in dB | BER for Stage 2 | | |
|---|---|---|---|---|
| | | MF Detector | CLPIC STAGE 2 | WLPIC STAGE 2 |
| 1 | -45 | 0.2642 | 0.2644 | 0.2660 |
| 2 | -40 | 0.1549 | 0.1547 | 0.1589 |
| 3 | -35 | 0.0707 | 0.0709 | 0.0693 |
| 4 | -30 | 0.0275 | 0.0271 | 0.0236 |
| 5 | -25 | 0.0109 | 0.0104 | 0.0083 |
| 6 | -20 | 0.0058 | 0.0052 | 0.0022 |
| 7 | -15 | 0.0048 | 0.0032 | 0.0011 |
| 8 | -10 | 0.0048 | 0.0029 | 0.0008 |

Table 3: $E_b/N_o$ Vs BER for Stage 3

| Sl. No | $E_b/N_o$ in dB | BER for Stage 3 | | |
|---|---|---|---|---|
| | | MF Detector | CLPIC STAGE 3 | WLPIC STAGE 3 |
| 1 | -45 | 0.2642 | 0.2649 | 0.2675 |
| 2 | -40 | 0.1549 | 0.1546 | 0.1561 |
| 3 | -35 | 0.0707 | 0.1546 | 0.1561 |
| 4 | -30 | 0.0275 | 0.0258 | 0.0238 |
| 5 | -25 | 0.0109 | 0.0096 | 0.0075 |
| 6 | -20 | 0.0058 | 0.0043 | 0.0020 |
| 7 | -15 | 0.0048 | 0.0026 | 0.0006 |
| 8 | -10 | 0.0048 | 0.0027 | 0.0003 |

From Table 2 and Table 3,We make interesting observations on the performance of the proposed WLPIC scheme. The proposed WLPIC scheme is shown to effectively cancel the MUI caused by the other user CFOs in uplink OFDMA and at the same time, WLPIC scheme (where the optimized weights derived are used) performs significantly better than CLPIC at the second and third stages. It is found that the bit error rate performance of the proposed weighted linear parallel interference cancellation is almost ten times better than that of the bit error performance of the conventional linear parallel interference at the final stage.

## VIII. Conclusion

The proposed scheme performed CFO compensation in time-domain, followed by 'K' DFT operations and multistage linear parallel interference cancellation on the DFT outputs. Estimates of the MUI for cancellation were obtained using soft values of the outputs from the previous stages. The MUI estimates were scaled by weights before cancellation. Expressions for the average SIR at the output of the 2nd and 3rd stages of the proposed scheme were derived. While these SIR expressions quantified the improvement in output SIR from one stage to the next, they were also used to obtain the optimum weights in-closed form. The proposed scheme was shown to effectively cancel the MUI caused by the other user CFOs and a comparison between our proposed scheme, WLPIC and the conventional system, CLPIC is also shown.

## References

[1] K. Kim, Y. Han, and S.-L. Kim, "Joint subcarrier and power allocation in uplink OFDMA systems," IEEE Communication Letter., vol. 9, no. 6, pp. 526-528, June 2005.

[2] Z. R. Cao, U. Tureli, and Y.-D. Yao, "Deterministic multiuser carrier frequency offset estimation for interleaved OFDMA uplink," IEEE Transaction on Communication, vol. 52, no. 9, pp. 1585-1594, Sep. 2004.

[3] H. Wang and B. Chen, "Asymptotic distributions and peak power analysis for uplink OFDMA," in Proc. IEEE ICASSP, May 2004, pp. iv-1085-8.

[4] M.O.Pun, J. Juo and M. Morelli, "Joint synchronization and channel estimation in uplink OFDMA systems," in Proc. IEEE ICASSP, Mar. 2005, vol. 3, pp. iii/857-iii/860.

[5] M. Tonello, N. Laurenti, and S. Pupolin, "Analysis of the uplink of an asynchronous multiuser DMT OFDMA system impaired by time offsets, frequency offsets, and multipath fading," in Proc. IEEE VTC (Fall), Oct. 2000, vol. 3, pp. 1094-1099.

[6] J. Choi, C. Lee, H. W. Jung, and Y. H. Lee, "Carrier frequency offset compensation for uplink of OFDM-FDMA systems," IEEE Communication Letter., vol. 4, no. 12, pp. 414-416, Dec. 2000.

[7] Z. Cao, U. Tureh, and Y. D. Yao, "Analysis of two receiver schemes for interleaved OFDMA uplink signals," in Proc. 36th Asilomar Conf. Signals, System and Computer., Nov. 2002, vol. 2, pp. 1818-1821.

[8] R. Fantacci, D. Marabissi, and S. Papini, "Multiuser interference cancellation receivers for OFDMA uplink communications with carrier frequency offset," in Proc. IEEE GLOBECOM, Nov.-Dec. 2004, pp. 2808-2812.

[9] D. Huang and K. B. Letaief, "An interference cancellation scheme for carrier frequency offsets correction in OFDMA systems," IEEE Trans. Commun., vol. 53, no. 7, pp. 1155-1165, July 2005.

[10] T. Pollet, M. V. Bladel, and M. Moeneclaey, "BER sensitivity of OFDM systems to carrier frequency offset and Weiner

phase noise," IEEE Trans. Commun., vol. 43, pp. 191-193, Feb./Mar./Apr. 1995.
[11] L. Rugini, P. Banelli, and S. Cacopardi, "Probability of error of OFDM systems with carrier offset in frequency-selective fading channels," in Proc. IEEE GLOBECOM, Nov.-Dec. 2004, pp. 3289-3293.
[12] D. Divsalar, M. K. Simon, and D. Raphaeli, "Improved parallel interference cancellation for CDMA," IEEE Transaction on Communication, vol. 46, no. 2, pp. 258-268, Feb. 1998.
[13] D. R. Brown, M. Motani, V. Veeravalli, H. V. Poor, and C. R. Johnson, Jr., "On the performance of linear parallel interference cancellation," IEEE Trans. Inform. Theory, vol. 47, no. 5, pp. 1957-1970, July 2001.
[14] D. Guo, L. K. Rasmussen, S. Sun, and T. J. Lim, "A matrix-algebraic approach to linear parallel interference cancellation in CDMA," IEEE Transaction on Communication., vol. 48, no. 1, pp. 152-161, Jan. 2000.



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